```
iter_via_intseg_shift_rw  THM

⊢  ∀A:Type, f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A), k:ℤ.
|  (Iter(f;u) i:{a..b⁻}. e(i)) = (Iter(f;u) j:{a+k..b+k⁻}. e(j-k))
| by Guarding (∀a:<type>. <prop>) Auto
|
1. A : Type
2. f : A→A→A
3. u : A
⊢  ∀a,b:ℤ, e:({a..b⁻}→A), k:ℤ.
|  (Iter(f;u) i:{a..b⁻}. e(i)) = (Iter(f;u) j:{a+k..b+k⁻}. e(j-k))
| by BackThru:
|    Thm*  ∀P:(ℤ→ℤ→Prop).
|          (∀a:ℤ, b:{...a}. P(a,b))
|          ⇒
|          (∀a:ℤ, b:{a+1...}. (∀c:{a..b⁻}. P(a,c)) ⇒ P(a,b))
|          ⇒
|          (∀a,b:ℤ. P(a,b))
|    {all_int_segs_by_upper_ind}
|
|\
| 4. a : ℤ
| 5. b : {...a}
| 6. e : {a..b⁻}→A
| 7. k : ℤ
| ⊢  (Iter(f;u) i:{a..b⁻}. e(i)) = (Iter(f;u) j:{a+k..b+k⁻}. e(j-k))
|   by Rewrite by
|      Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
|            b≤a ⇒ (Iter(f;u) i:{a..b⁻}. e(i)) = u
|      {iter_via_intseg_null}
|
.\
. 4. a : ℤ
. 5. b : {a+1...}
. 6. ∀c:{a..b⁻}, e:({a..c⁻}→A), k:ℤ.
. |  (Iter(f;u) i:{a..c⁻}. e(i)) = (Iter(f;u) j:{a+k..c+k⁻}. e(j-k))
. 7. e : {a..b⁻}→A
. 8. k : ℤ
. ⊢  (Iter(f;u) i:{a..b⁻}. e(i)) = (Iter(f;u) j:{a+k..b+k⁻}. e(j-k))
. | by Rewrite by
. |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
. |          a<b
. |          ⇒
. |          (Iter(f;u) i:{a..b⁻}. e(i))
. |          =
. |          f((Iter(f;u) i:{a..b-1⁻}. e(i)),e(b-1))
. |    {iter_via_intseg_split_last}
. |
. ⊢  f((Iter(f;u) i:{a..b-1⁻}. e(i)),e(b-1))
. |  =
. |  f((Iter(f;u) j:{a+k..b+k-1⁻}. e(j-k)),e(b+k-1-k))  by Analyze
. |
. ⊢  f(Iter(f;u) i:{a..b-1⁻}. e(i))
. |  =
. |  f(Iter(f;u) j:{a+k..b+k-1⁻}. e(j-k))  by Analyze
```

```
. |
. ⊢  (Iter(f;u) i:{a..b-1⁻}. e(i))
.    =
.    (Iter(f;u) j:{a+k..b+k-1⁻}. e(j-k))
.   by Rewrite by Hyp:6 Using:[b:= b | c:= b-1]
END
```