```
iter_via_intseg_wf  THM

⊢  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
|  (Iter(f;u) i:{a..b⁻}. e(i)) ∈ A  by 4 Times Analyze
|
1. A : Type
2. f : A→A→A
3. u : A
4. a : ℤ
⊢  ∀b:ℤ, e:({a..b⁻}→A). (Iter(f;u) i:{a..b⁻}. e(i)) ∈ A
| by ∀n:ℕ, b:ℤ.
|    b≤n+a ⇒ (∀e:({a..b⁻}→A). (Iter(f;u) i:{a..b⁻}. e(i)) ∈ A)
|    Asserted
|
|\
| .....assertion .....
| ⊢  ∀n:ℕ, b:ℤ.
| |  b≤n+a ⇒ (∀e:({a..b⁻}→A). (Iter(f;u) i:{a..b⁻}. e(i)) ∈ A)
| | by CompNatInd Concl
| |
| 5. n : ℕ
| 6. ∀n1:ℕ.
| |  n1<n
| |  ⇒
| |  (∀b:ℤ.
| |   b≤n1+a ⇒ (∀e:({a..b⁻}→A). (Iter(f;u) i:{a..b⁻}. e(i)) ∈ A))
| ⊢  ∀b:ℤ. b≤n+a ⇒ (∀e:({a..b⁻}→A). (Iter(f;u) i:{a..b⁻}. e(i)) ∈ A)
| | by UnivCD
| |
| 7. b : ℤ
| 8. b≤n+a
| 9. e : {a..b⁻}→A
| ⊢  (Iter(f;u) i:{a..b⁻}. e(i)) ∈ A
| | by Compute
| |      ⌈Iter(f;u) i:{a..b⁻}. e(i)⌉
| |      *⇒
| |      if a<_2b→ f((Iter(f;u) i:{a..b⁻}. e(i)),e(b-1)) else u fi
| |    THEN
| |    SplitITE Concl
| |
| .....truecase .....
| 10. a<b
| ⊢  f((Iter(f;u) i:{a..b-1⁻}. e(i)),e(b-1)) ∈ A  by 2 Times Analyze
| |
| ⊢  (Iter(f;u) i:{a..b-1⁻}. e(i)) ∈ A
|   by Use:[b-1-a] BackThru: Hyp:6
|
.\
. 5. ∀n:ℕ, b:ℤ.
. |  b≤n+a ⇒ (∀e:({a..b⁻}→A). (Iter(f;u) i:{a..b⁻}. e(i)) ∈ A)
. ⊢  ∀b:ℤ, e:({a..b⁻}→A). (Iter(f;u) i:{a..b⁻}. e(i)) ∈ A  by UnivCD
. |
. 6. b : ℤ
. 7. e : {a..b⁻}→A
. ⊢  (Iter(f;u) i:{a..b⁻}. e(i)) ∈ A
```

```
. | by Use:[if b<₂a→ 0 else b-a fi] BackThru: Hyp:5
. |
. |\
. | .....wf .....
. | ⊢  if b<₂a→ 0 else b-a fi ∈ ℕ  by SplitITE Concl
. |
. |\
. | .....wf .....
. | ⊢  b ∈ ℤ  by Auto
. |
. |\
. | ⊢  b≤if b<₂a→ 0 else b-a fi+a  by SplitITE Concl
. |
. .\
. . .....wf .....
. . ⊢  (λi.e(i)) ∈ {a..b⁻}→A  by Auto
END
```