```
sum_exponent  THM

⊢  ∀i:ℤ, x,y:ℕ. i↑x·i↑y = i↑(x+y)  by Analyze
|
1. i : ℤ
⊢  ∀x,y:ℕ. i↑x·i↑y = i↑(x+y)
| by ∀x,y:ℕ. x≤y ⇒ i↑x·i↑y = i↑(x+y)  Asserted
|
|\
| 2. x : ℕ
| 3. y : ℕ
| 4. x≤y
| ⊢  i↑x·i↑y = i↑(x+y)
| | by Rewrite by
| |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A), k:ℤ.
| |          (Iter(f;u) i:{a..b⁻}. e(i))
| |          =
| |          (Iter(f;u) j:{a+k..b+k⁻}. e(j-k))
| |    {iter_via_intseg_shift_rw}
| |    Using:[A:= ℤ |
| |           f:= λx,y. x·y |
| |           u:= 1 |
| |           a:= 0 |
| |           e(z):= i |
| |           b:= y |
| |           k:= x]
| |    THEN
| |    ArithSimp Concl
| |
| ⊢  i↑x·(Π j:{x..x+y⁻}. i) = i↑(x+y)
|   by BackThru:
|      Thm*  ∀a,c,b:ℤ, e:({a..b⁻}→ℤ).
|            a≤c
|            ⇒
|            c≤b
|            ⇒
|            (Π i:{a..b⁻}. e(i))
|            =
|            (Π i:{a..c⁻}. e(i))·(Π i:{c..b⁻}. e(i))
|      {mul_via_intseg_split_mid}
|
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. 2. ∀x,y:ℕ. x≤y ⇒ i↑x·i↑y = i↑(x+y)
. 3. x : ℕ
. 4. y : ℕ
. ⊢  i↑x·i↑y = i↑(x+y)  by Decide: x≤y
. |
. |\
. | 5. x≤y
. | ⊢  i↑x·i↑y = i↑(x+y)  by BackThru: Hyp:2
. |
. .\
. . 5. ¬x≤y
. . ⊢  i↑x·i↑y = i↑(x+y)  by Inst: Hyp:2 Using:[y | x]
END
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