```
lang_union_inters  THM

⊢  ∀L,M,N:LangOver(Alph). (L U (M ∩ N)) = ((L U M) ∩ (L U N))
| by Unfold ‘languages‘ 0 THEN Unfold ‘lang_eq‘ 0
|    THEN
|    Unfold ‘lang_union‘ 0
|    THEN
|    Unfold ‘lang_inters‘ 0
|    THEN
|    UnivCD
|
1. Alph : Type
2. L : Alph*→Prop
3. M : Alph*→Prop
4. N : Alph*→Prop
5. l : Alph*
⊢  (λl.L(l) ∨ (λl.M(l) & N(l))(l))(l)
|  ⟺
|  (λl.(λl.L(l) ∨ M(l))(l) & (λl.L(l) ∨ N(l))(l))(l)
| by Reduce 0 THEN GenUnivCD
|
|\
| 6. L(l) ∨ M(l) & N(l)
| ⊢  L(l) ∨ M(l)  by Analyze 6
| |
| |\
| | 6. L(l)
| | ⊢   by Sel 1 (Analyze 0)
| |
| .\
| . 6. M(l) & N(l)
| . ⊢   by Analyze 6 THEN Sel 2 (Analyze 0)
|
|\
| 6. L(l) ∨ M(l) & N(l)
| ⊢  L(l) ∨ N(l)  by Analyze 6
| |
| |\
| | 6. L(l)
| | ⊢   by Sel 1 (Analyze 0)
| |
| .\
| . 6. M(l) & N(l)
| . ⊢   by Analyze 6
| . |
| . 6. M(l)
| . 7. N(l)
| . ⊢   by Sel 2 (Analyze 0)
|
.\
. 6. L(l) ∨ M(l) & L(l) ∨ N(l)
. ⊢  L(l) ∨ M(l) & N(l)  by Analyze 6
. |
. 6. L(l) ∨ M(l)
. 7. L(l) ∨ N(l)
```

```
. ⊢   by Analyze 6
. |
. |\
. | 6. L(l)
. | 7. L(l) ∨ N(l)
. | ⊢   by Sel 1 (Analyze 0)
. |
. .\
. . 6. M(l)
. . 7. L(l) ∨ N(l)
. . ⊢   by Analyze 7
. . |
. . |\
. . | 7. L(l)
. . | ⊢   by Sel 1 (Analyze 0)
. . |
. . .\
. . . 7. N(l)
. . . ⊢   by Sel 2 (Analyze 0) THEN Analyze 0
END
```