```
select_listify_id  THM

⊢  ∀n:ℕ, f:(ℕn→T), i:ℕn. (f{ℕn})[i] = f(i)  by UnivCD
|
1. T : Type
2. n : ℕ
3. f : ℕn→T
4. i : ℕn
⊢  (f{ℕn})[i] = f(i)
| by Assert (∀j:{...n}. 0≤j ⇒ (∀i:{j..n⁻}. (f{j..n⁻})[(i-j)] = f(i)))
|    THENA
|    Thin 4
|
|\
| 3. f : ℕn→T
| ⊢  ∀j:{...n}. 0≤j ⇒ (∀i:{j..n⁻}. (f{j..n⁻})[(i-j)] = f(i))
| | by (BackThru
| |     Thm* ∀i:ℤ, E:({...i}→Prop).
| |          E(i)
| |          ⇒
| |          (∀k:{...i-1}. E(k+1) ⇒ E(k)) ⇒ (∀k:{...i}. E(k))
| |     {int_lower_ind})
| |    THEN
| |    UnivCD
| |
| |\
| | 4. 0≤n
| | 5. i : {n..n⁻}
| | ⊢  (f{n..n⁻})[(i-n)] = f(i)  by Arith
| |
| .\
| . 4. j : {...n-1}
| . 5. 0≤j+1 ⇒ (∀i:{(j+1)..n⁻}. (f{(j+1)..n⁻})[(i-(j+1))] = f(i))
| . 6. 0≤j
| . 7. i : {j..n⁻}
| . ⊢  (f{j..n⁻})[(i-j)] = f(i)  by RecCaseSplit ‘listify‘
| . |
| . |\
| . | .....truecase .....
| . | 8. n≤j
| . | ⊢  nil[(i-j)] = f(i)  by Arith
| . |
| . .\
| . . .....falsecase .....
| . . 8. j<n
| . . ⊢  (f(j).f{(j+1)..n⁻})[(i-j)] = f(i)  by Decide (i = j)
| . . |
| . . |\
| . . | 9. i = j
| . . | ⊢  (f(j).f{(j+1)..n⁻})[(i-j)] = f(i)
| . . | | by RWH
| . . | |    (LemmaC
| . . | |     Thm* ∀a:T, as:T List, i:ℤ. i≤0 ⇒ (a.as)[i] = a
| . . | |     {select_cons_hd})
| . . | |    0
```

```
| . . | |
| . . | ⊢  f(j) = f(i)  by (Assert (i = j)) THEN (RWH (HypC -1) 0)
| . . |
| . . .\
| . . . 9. ¬i = j
| . . . ⊢  (f(j).f{(j+1)..n⁻})[(i-j)] = f(i)
| . . . | by RWH
| . . . |    (LemmaC
| . . . |     Thm* ∀a:T, as:T List, i:ℤ.
| . . . |           0<i ⇒ i≤||as|| ⇒ (a.as)[i] = as[(i-1)]
| . . . |     {select_cons_tl})
| . . . |    0
| . . . |
| . . . ⊢  (f{(j+1)..n⁻})[(i-j-1)] = f(i)
| . . .   by (InstHyp [i] 5) THEN (OnMCls [0;-1] ArithSimp)
|
.\
. 5. ∀j:{...n}. 0≤j ⇒ (∀i:{j..n⁻}. (f{j..n⁻})[(i-j)] = f(i))
. ⊢  (f{ℕn})[i] = f(i)  by (InstHyp [0;i] 5) THEN (ArithSimp -1)
END
```