```
append_segment  THM

⊢  ∀as:T*, i:{0...||as||}, j:{i...||as||}, k:{j...||as||}.
|  ((as[i..j⁻]) @ (as[j..k⁻])) = (as[i..k⁻])  by RepD
|
1. T : Type
2. as : T*
3. i : {0...||as||}
4. j : {i...||as||}
5. k : {j...||as||}
⊢  ((as[i..j⁻]) @ (as[j..k⁻])) = (as[i..k⁻])  by Unfold ‘segment‘ 0
|
⊢  (firstn(j-i;nth_tl(i;as)) @ firstn(k-j;nth_tl(j;as)))
|  =
|  firstn(k-i;nth_tl(i;as))  by Let (x = j-i)
|
6. x : ℕ
7. x = j-i
⊢  (firstn(x;nth_tl(i;as)) @ firstn(k-j;nth_tl(j;as)))
|  =
|  firstn(k-i;nth_tl(i;as))  by Assert (j = x+i)
|
8. j = x+i
⊢   by RWH (HypC 8) 0
|
⊢  (firstn(x;nth_tl(i;as)) @ firstn(k-(x+i);nth_tl(x+i;as)))
|  =
|  firstn(k-i;nth_tl(i;as))  by Assert (k≥x+i)
|
9. k≥x+i
⊢   by Thin 8 THEN Thin 7 THEN NatInd 6
|
|\
| .....basecase .....
| 5. k : {j...||as||}
| ⊢  k≥0+i ⇒
| |  (firstn(0;nth_tl(i;as)) @ firstn(k-(0+i);nth_tl(0+i;as)))
| |  =
| |  firstn(k-i;nth_tl(i;as))  by ArithSimp 0
| |
| ⊢  k≥i ⇒
| |  (firstn(0;nth_tl(i;as)) @ firstn(-i+k;nth_tl(i;as)))
| |  =
| |  firstn(-i+k;nth_tl(i;as))
| | by RWH (AddrC [2;1] (RecUnfoldC ‘firstn‘)) 0 THEN Reduce 0
| |
| ⊢  k≥i ⇒
| |  ((Case of nth_tl(i;as); nil → nil ; a.as’ → nil)
| |   @ firstn(-i+k;nth_tl(i;as)))
| |  =
| |  firstn(-i+k;nth_tl(i;as))
| | by Assert ((Case of nth_tl(i;as); nil → nil ; a.as’ → nil) = nil)
| |
| |\
| | .....assertion .....
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| | ⊢  (Case of nth_tl(i;as); nil → nil ; a.as' → nil) = nil
| | | by Let (w = nth_tl(i;as))
| | |
| | 6. w : T*
| | 7. w = nth_tl(i;as)
| | ⊢  (Case of w; nil → nil ; a.as' → nil) = nil
| |   by Thin -1 THEN Analyze -1 THEN Reduce 0
| |
| .\
| . 6. (Case of nth_tl(i;as); nil → nil ; a.as' → nil) = nil
| . ⊢   by (RWH (HypC -1) 0) THEN (Reduce 0)
| . |
| . ⊢  k≥i ⇒ firstn(-i+k;nth_tl(i;as)) = firstn(-i+k;nth_tl(i;as))
| .   by Auto
|
.\
. .....upcase .....
. 6. x : ℤ
. 7. 0<x
. 8. k≥x-1+i ⇒
. |  (firstn(x-1;nth_tl(i;as)) @ firstn(k-(x-1+i);nth_tl(x-1+i;as)))
. |  =
. |  firstn(k-i;nth_tl(i;as))
. ⊢  k≥x+i ⇒
. |  (firstn(x;nth_tl(i;as)) @ firstn(k-(x+i);nth_tl(x+i;as)))
. |  =
. |  firstn(k-i;nth_tl(i;as))  by (Analyze 0) THEN (Analyze 8)
. |
. 8. k≥x+i
. 9. (firstn(x-1;nth_tl(i;as)) @ firstn(k-(x-1+i);nth_tl(x-1+i;as)))
. |  =
. |  firstn(k-i;nth_tl(i;as))
. ⊢  (firstn(x;nth_tl(i;as)) @ firstn(k-(x+i);nth_tl(x+i;as)))
. |  =
. |  firstn(k-i;nth_tl(i;as))  by (RWH (RevHypC -1) 0) THEN (Thin -1)
. |
. 8. k≥x+i
. ⊢  (firstn(x;nth_tl(i;as)) @ firstn(k-(x+i);nth_tl(x+i;as)))
. |  =
. |  (firstn(x-1;nth_tl(i;as)) @ firstn(k-(x-1+i);nth_tl(x-1+i;as)))
. | by Inst
. |    Thm* ∀x:ℕ, as:T*.
. |         x<||as|| ⇒ firstn(x+1;as) = (firstn(x;as) @ [as[x]])
. |    {firstn_lem1}
. |    [T;x-1;nth_tl(i;as)]
. |
. 9. firstn(x-1+1;nth_tl(i;as))
. |  =
. |  (firstn(x-1;nth_tl(i;as)) @ [nth_tl(i;as)[(x-1)]])
. ⊢   by (ArithSimp -1) THEN (RWH (HypC -1) 0) THEN (Thin -1)
. |
. 8. k≥x+i
. ⊢  ((firstn(-1+x;nth_tl(i;as)) @ [nth_tl(i;as)[(-1+x)]])
. |   @ firstn(k-(x+i);nth_tl(x+i;as)))
. |  =
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. |  (firstn(x-1;nth_tl(i;as)) @ firstn(k-(x-1+i);nth_tl(x-1+i;as)))
. | by (ArithSimp 0)
. |    THEN
. |    (RWH
. |     (LemmaC
. |      Thm* ∀as,bs,cs:T*. ((as @ bs) @ cs) = (as @ (bs @ cs))
. |      {append_assoc})
. |     0)
. |
. ⊢  (firstn(-1+x;nth_tl(i;as))
. |   @ [nth_tl(i;as)[(-1+x)]]
. |   @ firstn(-i+k+-x;nth_tl(i+x;as)))
. |  =
. |  (firstn(-1+x;nth_tl(i;as)) @ firstn(1+-i+k+-x;nth_tl(-1+i+x;as)))
. | by Reduce 0 THEN Analyze
. |
. ⊢  nth_tl(i;as)[(-1+x)].firstn(-i+k+-x;nth_tl(i+x;as))
. |  =
. |  firstn(1+-i+k+-x;nth_tl(-1+i+x;as))  by Unfold ‘select‘ 0
. |
. ⊢  hd(nth_tl(-1+x;nth_tl(i;as))).firstn(-i+k+-x;nth_tl(i+x;as))
. |  =
. |  firstn(1+-i+k+-x;nth_tl(-1+i+x;as))
. | by Inst
. |    Thm* ∀x,y:ℕ, as:T*. nth_tl(x;nth_tl(y;as)) = nth_tl(x+y;as)
. |    {nth_tl_lem1}
. |    [T;-1+x;i;as]
. |
. 9. nth_tl(-1+x;nth_tl(i;as)) = nth_tl(-1+x+i;as)
. ⊢   by (RWH (HypC -1) 0) THEN (Thin -1)
. |
. 8. k≥x+i
. ⊢  hd(nth_tl(-1+x+i;as)).firstn(-i+k+-x;nth_tl(i+x;as))
. |  =
. |  firstn(1+-i+k+-x;nth_tl(-1+i+x;as))  by ArithSimp 0
. |
. ⊢  hd(nth_tl(-1+i+x;as)).firstn(-i+k+-x;nth_tl(i+x;as))
. |  =
. |  firstn(1+-i+k+-x;nth_tl(-1+i+x;as))
. | by (Inst
. |     Thm* ∀y:ℕ, as:T*. tl(nth_tl(y;as)) = nth_tl(y+1;as)
. |     {nth_tl_lem0}
. |     [T;i+x-1;as])
. |    THEN
. |    (ArithSimp -1)
. |
. 9. tl(nth_tl(-1+i+x;as)) = nth_tl(i+x;as)
. ⊢   by RWH (RevHypC -1) 0
. |
. ⊢  hd(nth_tl(-1+i+x;as)).firstn(-i+k+-x;tl(nth_tl(-1+i+x;as)))
. |  =
. |  firstn(1+-i+k+-x;nth_tl(-1+i+x;as))
. | by Let (aas = nth_tl(-1+i+x;as))
. |
. 10. aas : T*
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. 11. aas = nth_tl(-1+i+x;as)
. ⊢  hd(aas).firstn(-i+k+-x;tl(aas)) = firstn(1+-i+k+-x;aas)
. | by Assert (¬aas = nil)
. |
. |\
. | .....assertion .....
. | ⊢  ¬aas = nil
. | | by BackThru
. | |    Thm* ∀as:A*. ¬as = nil ⟺ ||as||≥1    {length_of_not_nil}
. | |
. | ⊢  ||aas||≥1  by RWH (HypC -1) 0
. |
. .\
. . 12. ¬aas = nil
. . ⊢   by Thin -2 THEN ListInd 10
. . |
. . |\
. . | 10. T*
. . | ⊢  ¬nil = nil ⇒
. . |    hd(nil).firstn(-i+k+-x;tl(nil)) = firstn(1+-i+k+-x;nil)
. . |   by Analyze 0 THEN Analyze -1
. . |
. . .\
. . . 11. u : T
. . . 12. v : T*
. . . 13. ¬v = nil ⇒
. . . |   hd(v).firstn(-i+k+-x;tl(v)) = firstn(1+-i+k+-x;v)
. . . ⊢  ¬u.v = nil ⇒
. . . |  hd((u.v)).firstn(-i+k+-x;tl((u.v))) = firstn(1+-i+k+-x;u.v)
. . . | by (Thin -1 THEN Analyze 0) THEN (Thin -1)
. . . |
. . . 12. v : T*
. . . ⊢  hd((u.v)).firstn(-i+k+-x;tl((u.v))) = firstn(1+-i+k+-x;u.v)
. . .   by RWH (AddrC [3] (RecUnfoldC ‘firstn‘)) 0 THEN Reduce 0
. . .      THEN
. . .      SplitOnConclITE
END
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