append_functionality_wrt_sublist THM

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⊢  ∀L1,L2:T List, eq1,eq2,eq3:{T≡₂}, L3,L4:T List.
|  Discrete{T}
|  ⇒
|  eq1 = eq2
|  ⇒
|  eq2 = eq3 ⇒ L1(⊆eq1)L2 ⇒ L3(⊆eq2)L4 ⇒ (L1 @ L3)(⊆eq3)(L2 @ L4)
| by RepeatFor 3 (Analyze 0)
|    THEN
|    RepeatFor 3 (Analyze 0 THEN Equivalence -1)
|    THEN
|    UnivCD
|
1. T : Type
2. L1 : T List
3. L2 : T List
4. eq1 : {T≡₂}
5. eq1 ∈ T→T→𝔹
6. ∀x:T. eq1(x,x)
7. ∀x,y:T. eq1(x,y) ⟺ eq1(y,x)
8. ∀x,y,z:T. eq1(x,y) ⇒ eq1(y,z) ⇒ eq1(x,z)
9. eq2 : {T≡₂}
10. eq2 ∈ T→T→𝔹
11. ∀x:T. eq2(x,x)
12. ∀x,y:T. eq2(x,y) ⟺ eq2(y,x)
13. ∀x,y,z:T. eq2(x,y) ⇒ eq2(y,z) ⇒ eq2(x,z)
14. eq3 : {T≡₂}
15. eq3 ∈ T→T→𝔹
16. ∀x:T. eq3(x,x)
17. ∀x,y:T. eq3(x,y) ⟺ eq3(y,x)
18. ∀x,y,z:T. eq3(x,y) ⇒ eq3(y,z) ⇒ eq3(x,z)
19. L3 : T List
20. L4 : T List
21. Discrete{T}
22. eq1 = eq2
23. eq2 = eq3
24. L1(⊆eq1)L2
25. L3(⊆eq2)L4
⊢  (L1 @ L3)(⊆eq3)(L2 @ L4)
| by HypSubst 22 24 THENA Equivalence -1 THEN HypSubst 23 24
|    THENA
|    Equivalence -1
|    THEN
|    HypSubst 23 25
|    THENA
|    Equivalence -1
|    THEN
|    RenameVar ‘eq’ 14
|    THEN
|    OnHyps [23;22;13;12;11;10;9;8;7;6;5;4] (λi.Thin i)
|
4. eq : {T≡₂}
5. eq ∈ T→T→𝔹
6. ∀x:T. eq(x,x)
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7. ∀x,y:T. eq(x,y) ⟺ eq(y,x)
8. ∀x,y,z:T. eq(x,y) ⇒ eq(y,z) ⇒ eq(x,z)
9. L3 : T List
10. L4 : T List
11. Discrete{T}
12. L1(⊆eq)L2
13. L3(⊆eq)L4
⊢  (L1 @ L3)(⊆eq)(L2 @ L4)  by ListInd 2
|
|\
| 12. L3(⊆eq)L4
| ⊢  nil(⊆eq)L2 ⇒ (nil @ L3)(⊆eq)(L2 @ L4)  by Reduce 0
| |
| ⊢  True ⇒ L3(⊆eq)(L2 @ L4)  by Analyze 0 THEN Thin -1
| |
| ⊢  L3(⊆eq)(L2 @ L4)  by ListInd -4 THEN Reduce 0
| |
| |\
| | 11. Discrete{T}
| | ⊢  True ⇒ True  by Auto
| |
| .\
| . 12. u : T
| . 13. v : T List
| . 14. v(⊆eq)L4 ⇒ v(⊆eq)(L2 @ L4)
| . ⊢  u(∈eq) L4 & v(⊆eq)L4 ⇒ u(∈eq) (L2 @ L4) & v(⊆eq)(L2 @ L4)
| . | by Auto
| . |
| . |\
| . | 15. u(∈eq) L4
| . | 16. v(⊆eq)L4
| . | ⊢  u(∈eq) (L2 @ L4)
| . | | by BackThru
| . | |    Thm* ∀eq:{T≡₂}, L,M:T List, x:T.
| . | |         x(∈eq) (L @ M) ⟺ x(∈eq) L ∨ x(∈eq) M
| . | |    {is_member_of_append_lemma}
| . | |
| . | ⊢  u(∈eq) L2 ∨ u(∈eq) L4  by Choose [2]
| . |
| . .\
| . . 15. u(∈eq) L4
| . . 16. v(⊆eq)L4
| . . ⊢  v(⊆eq)(L2 @ L4)  by HypBackchain
|
.\
. 12. L3(⊆eq)L4
. 13. u : T
. 14. v : T List
. 15. v(⊆eq)L2 ⇒ (v @ L3)(⊆eq)(L2 @ L4)
. ⊢  (u.v)(⊆eq)L2 ⇒ ((u.v) @ L3)(⊆eq)(L2 @ L4)
. | by Reduce 0 THEN UnivCD THEN Analyze -1 THEN Analyze 0
. |
. |\
. | 16. u(∈eq) L2
. | 17. v(⊆eq)L2
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. | ⊢  u(∈eq) (L2 @ L4)
. | | by BackThru
. | |    Thm* ∀eq:{T≡₂}, L,M:T List, x:T.
. | |         x(∈eq) (L @ M) ⟺ x(∈eq) L ∨ x(∈eq) M
. | |    {is_member_of_append_lemma}
. | |
. | ⊢  u(∈eq) L2 ∨ u(∈eq) L4  by Choose [1]
. |
. .\
. . 16. u(∈eq) L2
. . 17. v(⊆eq)L2
. . ⊢  (v @ L3)(⊆eq)(L2 @ L4)  by HypBackchain
END
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