```
decidable__list_exists  THM

⊢  ∀P:(T→Prop), L:T List. (∀x:T. Dec(P(x))) ⇒ Dec(∃x∈L.P(x))
| by UnivCD
|
1. T : Type
2. P : T→Prop
3. L : T List
4. ∀x:T. Dec(P(x))
⊢  Dec(∃x∈L.P(x))  by ListInd 3
|
|\
| ⊢  Dec(∃x∈nil.P(x))  by Rewrite (HigherC list_exists_unrollC) 0
| |
| ⊢  Dec(False)
|   by Unfold ‘decidable‘ 0 THEN DNth 2 0 THEN Unfold ‘not‘ 0
|
.\
. 5. u : T
. 6. v : T List
. 7. Dec(∃x∈v.P(x))
. ⊢  Dec(∃x∈(u.v).P(x))  by Rewrite (HigherC list_exists_unrollC) 0
. |
. ⊢  Dec(P(u) ∨ ∃x∈v.P(x))  by Unfold ‘decidable‘ 0
. |
. ⊢  P(u) ∨ ∃x∈v.P(x) ∨ ¬(P(u) ∨ ∃x∈v.P(x))  by Witness4 u
. |
. 4. u : T
. 5. v : T List
. 6. Dec(∃x∈v.P(x))
. 7. Dec(P(u))
. ⊢  P(u) ∨ ∃x∈v.P(x) ∨ ¬(P(u) ∨ ∃x∈v.P(x))
. | by Unfold ‘decidable‘ 7 THEN Analyze 7
. |
. |\
. | 7. P(u)
. | ⊢  P(u) ∨ ∃x∈v.P(x) ∨ ¬(P(u) ∨ ∃x∈v.P(x))
. |   by DNth 1 0 THEN DNth 1 0
. |
. .\
. . 7. ¬P(u)
. . ⊢  P(u) ∨ ∃x∈v.P(x) ∨ ¬(P(u) ∨ ∃x∈v.P(x))
. . | by Unfold ‘decidable‘ 6 THEN Analyze 6
. . |
. . |\
. . | 6. ∃x∈v.P(x)
. . | 7. ¬P(u)
. . | ⊢  P(u) ∨ ∃x∈v.P(x) ∨ ¬(P(u) ∨ ∃x∈v.P(x))
. . |   by DNth 1 0 THEN DNth 2 0
. . |
. . .\
. . . 6. ¬∃x∈v.P(x)
. . . 7. ¬P(u)
. . . ⊢  P(u) ∨ ∃x∈v.P(x) ∨ ¬(P(u) ∨ ∃x∈v.P(x))
. . . | by Unfold ‘not‘ 0 THEN DNth 2 0
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. . . |
. . . 8. P(u) ∨ ∃x∈v.P(x)
. . . ⊢  False  by OnHyps [6;7] (λi.Unfold ‘not‘ i) THEN Analyze 8
. . . |
. . . |\
. . . | 6. ∃x∈v.P(x) ⇒ False
. . . | 7. P(u) ⇒ False
. . . | 8. P(u)
. . . | ⊢  False  by FwdThru 7 [8]
. . . |
. . . .\
. . . . 6. ∃x∈v.P(x) ⇒ False
. . . . 7. P(u) ⇒ False
. . . . 8. ∃x∈v.P(x)
. . . . ⊢  False  by FwdThru 6 [8]
END
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