```
discrete__list  THM

⊢  Discrete{T} ⇒ Discrete{(T List)}  by UnivCD
|
1. T : Type
2. Discrete{T}
⊢  Discrete{(T List)}  by Analyze 0
|
3. x : T List
⊢  ∀y:T List. Dec(x = y)  by ListInd 3
|
|\
| ⊢  ∀y:T List. Dec(nil = y)  by Analyze 0 THEN UnfoldTopAb 0
| |
| 4. y : T List
| ⊢  nil = y ∨ ¬nil = y  by Analyze 4
| |
| |\
| | ⊢  nil = nil ∨ ¬nil = nil  by DNth 1 0
| |
| .\
| . 5. u : T
| . 6. v : T List
| . ⊢  nil = u.v ∨ ¬nil = u.v  by DNth 2 0 THEN Analyze 0
| . |
| . 7. nil = u.v
| . ⊢  False
| . | by ApFunToHypEquands ‘L’ (|·|(L)) ℕ -1 THEN Reduce -1
| . |    THEN
| . |    MoveToConcl -1
| . |
| . ⊢  0 = 1+|·|(v) ⇒ False  by GenConcl (|·|(v) = n)
|
.\
. 4. u : T
. 5. v : T List
. 6. ∀y:T List. Dec(v = y)
. ⊢  ∀y:T List. Dec(u.v = y)  by Analyze 0 THEN ListInd -1
. |
. |\
. | 7. T List
. | ⊢  Dec(u.v = nil)  by UnfoldTopAb 0 THEN DNth 2 0
. | |
. | ⊢  ¬u.v = nil  by Analyze 0
. | |
. | 8. u.v = nil
. | ⊢  False
. |   by ApFunToHypEquands ‘L’ (|·|(L)) ℤ 8 THEN AbReduce -1
. |      THEN
. |      Inst Thm* ∀L:T List. |·|(L)≥0    {list_length_non_negative}
. |      [T;v]
. |
. .\
. . 7. T List
. . 8. u1 : T
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. . 9. v1 : T List
. . 10. Dec(u.v = v1)
. . ⊢  Dec(u.v = u1.v1)  by Analyze 10
. . |
. . |\
. . | 10. u.v = v1
. . | ⊢  Dec(u.v = u1.v1)  by Unfold ‘decidable‘ 0 THEN DNth 2 0
. . | |
. . | ⊢  ¬u.v = u1.v1  by Subst (v1 = u.v) 0
. . | |
. . | ⊢  ¬u.v = [u1; u/ v]  by Analyze 0
. . | |
. . | 11. u.v = [u1; u/ v]
. . | ⊢  False
. . |   by ApFunToHypEquands ‘L’ (|·|(L)) ℤ 11 THEN AbReduce -1
. . |      THEN
. . |      Inst Thm* ∀L:T List. |·|(L)≥0    {list_length_non_negative}
. . |      [T;v]
. . |
. . .\
. . . 10. ¬u.v = v1
. . . ⊢  Dec(u.v = u1.v1)  by Unfold ‘decidable‘ 0
. . . |
. . . ⊢  u.v = u1.v1 ∨ ¬u.v = u1.v1  by Witness6 v1
. . . |
. . . 6. T List
. . . 7. u1 : T
. . . 8. v1 : T List
. . . 9. ¬u.v = v1
. . . 10. Dec(v = v1)
. . . ⊢  u.v = u1.v1 ∨ ¬u.v = u1.v1  by Analyze 10
. . . |
. . . |\
. . . | 10. v = v1
. . . | ⊢  u.v = u1.v1 ∨ ¬u.v = u1.v1
. . . | | by FwdThru
. . . | |    Thm* Discrete{T}
. . . | |         ⇒
. . . | |         (∃f:(T→T→𝔹). ∀x,y:T. f(x,y) ⟺ x = y)
. . . | |    {discrete_implies_bool_equality}
. . . | |    [2]
. . . | |    THEN
. . . | |    Analyze -1
. . . | |
. . . | 11. f : T→T→𝔹
. . . | 12. ∀x,y:T. f(x,y) ⟺ x = y
. . . | ⊢  u.v = u1.v1 ∨ ¬u.v = u1.v1
. . . | | by With u (Analyze -1) THEN With u1 (Analyze -1)
. . . | |    THEN
. . . | |    Decide (f(u,u1))
. . . | |
. . . | |\
. . . | | 12. f(u,u1) ⟺ u = u1
. . . | | 13. f(u,u1)
. . . | | ⊢  u.v = u1.v1 ∨ ¬u.v = u1.v1  by DNth 1 0
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. . . | | |
. . . | | ⊢  u.v = u1.v1  by FwdThru 12 [13]
. . . | | |
. . . | | 14. u = u1
. . . | | ⊢  u.v = u1.v1  by MemberEqCD THEN Trivial
. . . | |
. . . | .\
. . . | . 12. f(u,u1) ⟺ u = u1
. . . | . 13. ¬f(u,u1)
. . . | . ⊢  u.v = u1.v1 ∨ ¬u.v = u1.v1  by DNth 2 0
. . . | . |
. . . | . ⊢  ¬u.v = u1.v1  by UnfoldTopAb 0 THEN Analyze 0
. . . | . |
. . . | . 14. u.v = u1.v1
. . . | . ⊢  False
. . . | . | by ApFunToHypEquands ‘L’ (Case of L; nil → u ; h.t → h) T
. . . | . |    14
. . . | . |
. . . | . 15. (Case of u.v; nil → u ; h.t → h)
. . . | . |    =
. . . | . |    (Case of u1.v1; nil → u ; h.t → h)
. . . | . ⊢  False  by Reduce -1
. . . | . |
. . . | . 15. u = u1
. . . | . ⊢  False  by Analyze -3
. . . | . |
. . . | . 13. u.v = u1.v1
. . . | . 14. u = u1
. . . | . ⊢  f(u,u1)  by HypBackchain
. . . |
. . . .\
. . . . 10. ¬v = v1
. . . . ⊢  u.v = u1.v1 ∨ ¬u.v = u1.v1
. . . . | by FwdThru
. . . . |    Thm* Discrete{T}
. . . . |         ⇒
. . . . |         (∃f:(T→T→𝔹). ∀x,y:T. f(x,y) ⟺ x = y)
. . . . |    {discrete_implies_bool_equality}
. . . . |    [2]
. . . . |    THEN
. . . . |    Analyze -1
. . . . |
. . . . 11. f : T→T→𝔹
. . . . 12. ∀x,y:T. f(x,y) ⟺ x = y
. . . . ⊢  u.v = u1.v1 ∨ ¬u.v = u1.v1
. . . . | by With u (Analyze -1) THEN With u1 (Analyze -1)
. . . . |    THEN
. . . . |    Decide (f(u,u1))
. . . . |
. . . . |\
. . . . | 12. f(u,u1) ⟺ u = u1
. . . . | 13. f(u,u1)
. . . . | ⊢  u.v = u1.v1 ∨ ¬u.v = u1.v1  by DNth 2 0
. . . . | |
. . . . | ⊢  ¬u.v = u1.v1  by UnfoldTopAb 0 THEN Analyze 0
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. . . . | |
. . . . | 14. u.v = u1.v1
. . . . | ⊢  False
. . . . | | by ApFunToHypEquands ‘L’ (Case of L; nil → nil ; h.t → t)
. . . . | |    (T List)
. . . . | |    14
. . . . | |
. . . . | 15. (Case of u.v; nil → nil ; h.t → t)
. . . . | |   =
. . . . | |   (Case of u1.v1; nil → nil ; h.t → t)
. . . . | ⊢  False  by Reduce -1
. . . . | |
. . . . | 15. v = v1
. . . . | ⊢  False  by Analyze 10
. . . . |
. . . . .\
. . . . . 12. f(u,u1) ⟺ u = u1
. . . . . 13. ¬f(u,u1)
. . . . . ⊢  u.v = u1.v1 ∨ ¬u.v = u1.v1  by DNth 2 0
. . . . . |
. . . . . ⊢  ¬u.v = u1.v1  by UnfoldTopAb 0 THEN Analyze 0
. . . . . |
. . . . . 14. u.v = u1.v1
. . . . . ⊢  False
. . . . . | by ApFunToHypEquands ‘L’ (Case of L; nil → u ; h.t → h) T
. . . . . |    14
. . . . . |
. . . . . 15. (Case of u.v; nil → u ; h.t → h)
. . . . . |   =
. . . . . |   (Case of u1.v1; nil → u ; h.t → h)
. . . . . ⊢  False  by Reduce -1
. . . . . |
. . . . . 15. u = u1
. . . . . ⊢  False  by Analyze 13 THEN HypBackchain
END
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