```
is_intersection_implies_exists_element  THM

⊢  ∀eq:{T=₂}, L,M:T List. L∩(eq)M ⇒ (∃x:{x:T| x(∈eq) L }. x(∈eq) M)
| by UnivCD
|
1. T : Type
2. eq : {T=₂}
3. L : T List
4. M : T List
5. L∩(eq)M
⊢  ∃x:{x:T| x(∈eq) L }. x(∈eq) M  by ListInd 3 THEN Reduce 0
|
|\
| 4. M : T List
| ⊢  False ⇒ (∃x:{x:T| False }. x(∈eq) M)  by Auto
|
.\
. 5. u : T
. 6. v : T List
. 7. v∩(eq)M ⇒ (∃x:{x:T| x(∈eq) v }. x(∈eq) M)
. ⊢  u(∈eq) M ∨ ∃x∈v.x(∈eq) M
. |  ⇒
. |  (∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. x(∈eq) M)
. | by Analyze 0 THEN Analyze -1
. |
. |\
. | 8. u(∈eq) M
. | ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. x(∈eq) M
. | | by Witness u
. | |
. | .....wf .....
. | ⊢  u ∈ {x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }  by Analyze
. | |
. | .....set predicate .....
. | ⊢  if eq(u,u)→ true₂ else u(∈eq) v fi
. | | by Subst (eq(u,u) = true₂) 0 THENL [Id;Reduce 0 THEN Trivial]
. | |
. | .....equality .....
. | ⊢  eq(u,u) = true₂
. | | by BackThru Thm* ∀P:𝔹. P ⟺ P = true₂    {assert_iff_btrue}
. | |
. | ⊢  eq(u,u)
. |   by BackThru
. |      Thm* ∀eq:{T=₂}, u:T. eq(u,u)
. |      {discrete_equality_reflexive}
. |
. .\
. . 8. ∃x∈v.x(∈eq) M
. . ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. x(∈eq) M
. . | by Decide v∩(eq)M
. . |
. . |\
. . | 9. v∩(eq)M
. . | ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. x(∈eq) M
. . | | by FwdThru 7 [-1]
```

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. . | |
. . | 10. ∃x:{x:T| x(∈eq) v }. x(∈eq) M
. . | ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. x(∈eq) M
. . | | by Analyze -1
. . | |
. . | 10. x : {x:T| x(∈eq) v }
. . | 11. x(∈eq) M
. . | ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. x(∈eq) M
. . | | by With x (Analyze 0)
. . | |
. . | .....wf .....
. . | ⊢  x ∈ {x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }  by Analyze
. . | |
. . | .....set predicate .....
. . | ⊢  if eq(x,u)→ true₂ else x(∈eq) v fi  by IfThenElseCases 0
. . | |
. . | |\
. . | | .....truecase .....
. . | | 12. eq(x,u) = true₂
. . | | ⊢  true₂  by Reduce 0 THEN Trivial
. . | |
. . | .\
. . | . .....falsecase .....
. . | . 12. eq(x,u) = false₂
. . | . ⊢  x(∈eq) v  by Analyze 10 THEN UnhideSqStableHyp 11
. . |
. . .\
. . . 9. ¬v∩(eq)M
. . . ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. x(∈eq) M
. . . | by Fold `is_intersection` -2
. . . |
. . . 8. v∩(eq)M
. . . 9. ¬v∩(eq)M
. . . ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. x(∈eq) M
. . .   by Analyze -1 THEN Trivial
END
```