```
is_member_append_disjunction_lemma  THM

⊢  ∀eq:{T≡₂}, x:T, L,M:T List. x(∈eq) (L @ M) ⟺ x(∈eq) L ∨ x(∈eq) M
| by UnivCD
|
1. T : Type
2. eq : {T≡₂}
3. x : T
4. L : T List
5. M : T List
⊢  x(∈eq) (L @ M) ⟺ x(∈eq) L ∨ x(∈eq) M  by Analyze 0 THEN Analyze 0
|
|\
| 6. x(∈eq) (L @ M)
| ⊢  x(∈eq) L ∨ x(∈eq) M  by ListInd 4 THEN Reduce 0
| |
| |\
| | 5. M : T List
| | ⊢  x(∈eq) M ⇒ False ∨ x(∈eq) M  by Auto
| | |
| | 6. x(∈eq) M
| | ⊢  False ∨ x(∈eq) M  by DNth 2 0
| |
| .\
| . 6. u : T
| . 7. v : T List
| . 8. x(∈eq) (v @ M) ⇒ x(∈eq) v ∨ x(∈eq) M
| . ⊢  if eq(x,u)→ true₂ else x(∈eq) (v @ M) fi
| . |  ⇒
| . |  if eq(x,u)→ true₂ else x(∈eq) v fi ∨ x(∈eq) M  by Analyze 0
| . |
| . 9. if eq(x,u)→ true₂ else x(∈eq) (v @ M) fi
| . ⊢  if eq(x,u)→ true₂ else x(∈eq) v fi ∨ x(∈eq) M
| . | by IfThenElseCases 0 THEN Reduce 0
| . |
| . |\
| . | 10. eq(x,u) = true₂
| . | ⊢  True ∨ x(∈eq) M  by DNth 1 0
| . |
| . .\
| . . .....falsecase .....
| . . 10. eq(x,u) = false₂
| . . ⊢  x(∈eq) v ∨ x(∈eq) M  by HypSubst -1 -2 THEN Reduce -2
| . . |
| . . 9. x(∈eq) (v @ M)
| . . 10. eq(x,u) = false₂
| . . ⊢  x(∈eq) v ∨ x(∈eq) M  by HypBackchain
|
.\
. 6. x(∈eq) L ∨ x(∈eq) M
. ⊢  x(∈eq) (L @ M)  by ListInd 4 THEN Reduce 0
. |
. |\
. | 5. M : T List
. | ⊢  False ∨ x(∈eq) M ⇒ x(∈eq) M  by Analyze 0 THEN Analyze -1
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. .\
. . 6. u : T
. . 7. v : T List
. . 8. x(∈eq) v ∨ x(∈eq) M ⇒ x(∈eq) (v @ M)
. . ⊢  if eq(x,u)→ true₂ else x(∈eq) v fi ∨ x(∈eq) M
. . |  ⇒
. . |  if eq(x,u)→ true₂ else x(∈eq) (v @ M) fi  by Analyze 0
. . |
. . 9. if eq(x,u)→ true₂ else x(∈eq) v fi ∨ x(∈eq) M
. . ⊢  if eq(x,u)→ true₂ else x(∈eq) (v @ M) fi
. . | by IfThenElseCases 0 THEN Reduce 0
. . |
. . |\
. . | 10. eq(x,u) = true₂
. . | ⊢  True  by Trivial
. . |
. . .\
. . . .....falsecase .....
. . . 10. eq(x,u) = false₂
. . . ⊢  x(∈eq) (v @ M)  by HypBackchain
. . . |
. . . ⊢  x(∈eq) v ∨ x(∈eq) M  by HypSubst -1 -2 THEN Reduce -2
. . . |
. . . 9. x(∈eq) v ∨ x(∈eq) M
. . . 10. eq(x,u) = false₂
. . . ⊢  x(∈eq) v ∨ x(∈eq) M  by Trivial
END
```