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is_member_equalities_lemma  THM

⊢  ∀eq:{T=₂}, u:T, L:T List. u(∈eq) L ⇒ (∀f:{T≡₂}. u(∈f) L)
| by Analyze 0 THEN Analyze 0 THEN DiscreteEq -1 THEN UnivCD
|    THEN
|    Equivalence -1
|
1. T : Type
2. eq : {T=₂}
3. eq ∈ T→T→𝔹
4. ∀x,y:T. eq(x,y) ⟺ x = y
5. u : T
6. L : T List
7. u(∈eq) L
8. f : {T≡₂}
9. f ∈ T→T→𝔹
10. ∀x:T. f(x,x)
11. ∀x,y:T. f(x,y) ⟺ f(y,x)
12. ∀x,y,z:T. f(x,y) ⇒ f(y,z) ⇒ f(x,z)
⊢  u(∈f) L  by ListInd 6 THEN Rewrite (HigherC is_member_unrollC) 0
|
|\
| 7. f : {T≡₂}
| 8. f ∈ T→T→𝔹
| 9. ∀x:T. f(x,x)
| 10. ∀x,y:T. f(x,y) ⟺ f(y,x)
| 11. ∀x,y,z:T. f(x,y) ⇒ f(y,z) ⇒ f(x,z)
| ⊢  false₂ ⇒ false₂  by Rewrite (HigherC assert_evalC) 0
|
.\
. 7. f : {T≡₂}
. 8. f ∈ T→T→𝔹
. 9. ∀x:T. f(x,x)
. 10. ∀x,y:T. f(x,y) ⟺ f(y,x)
. 11. ∀x,y,z:T. f(x,y) ⇒ f(y,z) ⇒ f(x,z)
. 12. u1 : T
. 13. v : T List
. 14. u(∈eq) v ⇒ u(∈f) v
. ⊢  if eq(u,u1)→ true₂ else u(∈eq) v fi
. |  ⇒
. |  if f(u,u1)→ true₂ else u(∈f) v fi
. | by GenConcl (eq(u,u1) = b) THEN BoolInd -2
. |    THEN
. |    Rewrite
. |    (HigherC ifthenelse_evalC THENC TryC (HigherC assert_evalC))
. |    0
. |    THEN
. |    Analyze 0
. |
. |\
. | 15. eq(u,u1) = true₂
. | 16. True
. | ⊢  if f(u,u1)→ true₂ else u(∈f) v fi
. | | by FwdThru Thm* ∀P:𝔹. P ⟺ P = true₂    {assert_iff_btrue} [15]
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. | 17. eq(u,u1)
. | ⊢  if f(u,u1)→ true₂ else u(∈f) v fi  by FwdThru 4 [-1]
. | |
. | 18. u = u1
. | ⊢  if f(u,u1)→ true₂ else u(∈f) v fi
. | | by HypSubst 18 0 THEN Witness9 u1
. | |    THEN
. | |    FwdThru Thm* ∀P:𝔹. P ⟺ P = true₂    {assert_iff_btrue} [-1]
. | |
. | 9. ∀x,y:T. f(x,y) ⟺ f(y,x)
. | 10. ∀x,y,z:T. f(x,y) ⇒ f(y,z) ⇒ f(x,z)
. | 11. u1 : T
. | 12. v : T List
. | 13. u(∈eq) v ⇒ u(∈f) v
. | 14. eq(u,u1) = true₂
. | 15. True
. | 16. eq(u,u1)
. | 17. u = u1
. | 18. f(u1,u1)
. | 19. f(u1,u1) = true₂
. | ⊢  if f(u1,u1)→ true₂ else u1(∈f) v fi
. |   by HypSubst 19 0
. |      THEN
. |      Rewrite (HigherC ifthenelse_evalC THENC assert_evalC) 0
. |      THEN
. |      Trivial
. |
. .\
. . 15. eq(u,u1) = false₂
. . 16. u(∈eq) v
. . ⊢  if f(u,u1)→ true₂ else u(∈f) v fi
. . | by GenConcl (f(u,u1) = b) THEN BoolInd -2
. . |    THEN
. . |    Rewrite
. . |    (HigherC ifthenelse_evalC THENC TryC (HigherC assert_evalC))
. . |    0
. . |    THEN
. . |    Try Trivial
. . |
. . 17. f(u,u1) = false₂
. . ⊢  u(∈f) v  by BackThru 14 THEN Trivial
END
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