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list_all_append_lemma  THM

⊢  ∀P:(T→Prop), L,M:T List. ∀x∈(L @ M).P(x) ⟺ ∀x∈L.P(x) & ∀x∈M.P(x)
| by UnivCD THEN Analyze 0 THEN Analyze 0
|
|\
| 1. T : Type
| 2. P : T→Prop
| 3. L : T List
| 4. M : T List
| 5. ∀x∈(L @ M).P(x)
| ⊢  ∀x∈L.P(x) & ∀x∈M.P(x)
| | by MoveToConcl 4 THEN ListInd 3 THEN Reduce 0
| |
| |\
| | 3. T List
| | ⊢  ∀M:T List. ∀x∈M.P(x) ⇒ True & ∀x∈M.P(x)  by Auto
| |
| .\
| . 4. u : T
| . 5. v : T List
| . 6. ∀M:T List. ∀x∈(v @ M).P(x) ⇒ ∀x∈v.P(x) & ∀x∈M.P(x)
| . ⊢  ∀M:T List.
| . |  P(u) & ∀x∈(v @ M).P(x) ⇒ P(u) & ∀x∈v.P(x) & ∀x∈M.P(x)
| . | by UnivCD THEN Analyze -1 THEN Analyze 0
| . |
| . |\
| . | 7. M : T List
| . | 8. P(u)
| . | 9. ∀x∈(v @ M).P(x)
| . | ⊢  P(u) & ∀x∈v.P(x)  by Analyze 0
| . | |
| . | |\
| . | | ⊢  P(u)  by Auto
| . | |
| . | .\
| . | . ⊢  ∀x∈v.P(x)  by FwdThru 6 [-1]
| . | . |
| . | . 10. ∀x∈v.P(x)
| . | . ⊢  ∀x∈v.P(x)  by Auto
| . |
| . .\
| . . 7. M : T List
| . . 8. P(u)
| . . 9. ∀x∈(v @ M).P(x)
| . . ⊢  ∀x∈M.P(x)  by Sel 2 (FwdThru 6 [-1])
| . . |
| . . 10. ∀x∈M.P(x)
| . . ⊢  ∀x∈M.P(x)  by Auto
|
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. 1. T : Type
. 2. P : T→Prop
. 3. L : T List
. 4. M : T List
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. 5. ∀x∈L.P(x) & ∀x∈M.P(x)
. ⊢  ∀x∈(L @ M).P(x)  by Analyze -1
. |
. 5. ∀x∈L.P(x)
. 6. ∀x∈M.P(x)
. ⊢  ∀x∈(L @ M).P(x)  by MoveToConcl 4 THEN ListInd 3 THEN Reduce 0
. |
. |\
. | 3. T List
. | ⊢  True ⇒ (∀M:T List. ∀x∈M.P(x) ⇒ ∀x∈M.P(x))  by Auto
. |
. .\
. . 4. u : T
. . 5. v : T List
. . 6. ∀x∈v.P(x) ⇒ (∀M:T List. ∀x∈M.P(x) ⇒ ∀x∈(v @ M).P(x))
. . ⊢  P(u) & ∀x∈v.P(x)
. . |  ⇒
. . |  (∀M:T List. ∀x∈M.P(x) ⇒ P(u) & ∀x∈(v @ M).P(x))  by Auto
. . |
. . 7. P(u)
. . 8. ∀x∈v.P(x)
. . 9. M : T List
. . 10. ∀x∈M.P(x)
. . ⊢  ∀x∈(v @ M).P(x)  by HypBackchain
END
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