```
list_exists_exists  THM

⊢  ∀eq:{T=₂}, P:(T→Type), L:T List.
|  ∃x∈L.P(x) ⟺ (∃x:{x:T| x(∈eq) L }. P(x))  by UnivCD
|
1. T : Type
2. eq : {T=₂}
3. P : T→Type
4. L : T List
⊢  ∃x∈L.P(x) ⟺ (∃x:{x:T| x(∈eq) L }. P(x))
| by ListInd -1 THEN AbReduce 0
|
|\
| ⊢  False ⟺ (∃x:{x:T| False }. P(x))
| | by Auto THEN OnHyps [-1;-2] Analyze
| |
| 5. x : T
| 6. False
| 7. P(x)
| ⊢  False  by Trivial
|
.\
. 5. u : T
. 6. v : T List
. 7. ∃x∈v.P(x) ⟺ (∃x:{x:T| x(∈eq) v }. P(x))
. ⊢  P(u) ∨ ∃x∈v.P(x)
. |  ⟺
. |  (∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. P(x))  by Auto
. |
. |\
. | 7. ∃x∈v.P(x) ⇒ (∃x:{x:T| x(∈eq) v }. P(x))
. | 8. ∃x∈v.P(x) ⇐ (∃x:{x:T| x(∈eq) v }. P(x))
. | 9. P(u) ∨ ∃x∈v.P(x)
. | ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. P(x)
. | | by Analyze -1
. | |
. | |\
. | | 9. P(u)
. | | ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. P(x)
. | | | by Witness u THEN AbReduce 0
. | | |
. | | |\
. | | | .....wf .....
. | | | ⊢  u ∈ {x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }
. | | | | by AbSetMemEqTypeCD THEN AbReduce 0
. | | | |
. | | | .....set predicate .....
. | | | ⊢  if eq(u,u)→ true₂ else u(∈eq) v fi  by DiscreteEq 2
. | | | |
. | | | 3. eq ∈ T→T→𝔹
. | | | 4. ∀x,y:T. eq(x,y) ⟺ x = y
. | | | 5. P : T→Type
. | | | 6. T List
. | | | 7. u : T
. | | | 8. v : T List
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. | | | 9. ∃x∈v.P(x) ⇒ (∃x:{x:T| x(∈eq) v }. P(x))
. | | | 10. ∃x∈v.P(x) ⇐ (∃x:{x:T| x(∈eq) v }. P(x))
. | | | 11. P(u)
. | | | ⊢  if eq(u,u)→ true₂ else u(∈eq) v fi  by IfThenElseCases 0
. | | | |
. | | | |\
. | | | | .....truecase .....
. | | | | 12. eq(u,u) = true₂
. | | | | ⊢  true₂
. | | | |   by BackThru
. | | | |      Thm* ∀P:𝔹. P ⟺ P = true₂    {assert_iff_btrue}
. | | | |
. | | | .\
. | | | . .....falsecase .....
. | | | . 12. eq(u,u) = false₂
. | | | . ⊢  u(∈eq) v  by Witness4 u THEN With u (Analyze -1)
. | | | . |
. | | | . 4. P : T→Type
. | | | . 5. T List
. | | | . 6. u : T
. | | | . 7. v : T List
. | | | . 8. ∃x∈v.P(x) ⇒ (∃x:{x:T| x(∈eq) v }. P(x))
. | | | . 9. ∃x∈v.P(x) ⇐ (∃x:{x:T| x(∈eq) v }. P(x))
. | | | . 10. P(u)
. | | | . 11. eq(u,u) = false₂
. | | | . 12. eq(u,u) ⟺ u = u
. | | | . ⊢  u(∈eq) v
. | | | . | by FwdThru
. | | | . |    Thm* ∀P:𝔹. ¬P ⟺ P = false₂    {not_assert_iff_bfalse}
. | | | . |    [-2]
. | | | . |
. | | | . 12. eq(u,u) ⇒ u = u
. | | | . 13. (eq(u,u)) ⇐ (u = u)
. | | | . 14. ¬eq(u,u)
. | | | . ⊢  u(∈eq) v  by Analyze -1
. | | | . |
. | | | . 13. (eq(u,u)) ⇐ (u = u)
. | | | . ⊢  eq(u,u)  by HypBackchain
. | | |
. | | .\
. | | . ⊢  P(u)  by Trivial
. | |
. | .\
. | . 9. ∃x∈v.P(x)
. | . ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. P(x)
. | . | by FwdThru -3 [-1] THEN Analyze -1
. | . |

. | . 10. x : {x:T| x(∈eq) v }
. | . 11. P(x)
. | . ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. P(x)
. | . | by Witness x
. | . |
. | . |\
. | . | .....wf .....
. | . | ⊢  x ∈ {x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }
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. | . | | by AbSetMemEqTypeCD
. | . | |
. | . | |\
. | . | | ⊢  x ∈ T  by Analyze -2
. | . | |
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. | . | . .....set predicate .....
. | . | . ⊢  if eq(x,u)→ true₂ else x(∈eq) v fi  by IfThenElseCases 0
. | . | . |
. | . | . |\
. | . | . | .....truecase .....
. | . | . | 12. eq(x,u) = true₂
. | . | . | ⊢  true₂
. | . | . |   by BackThru
. | . | . |      Thm* ∀P:𝔹. P ⟺ P = true₂    {assert_iff_btrue}
. | . | . |
. | . | . .\
. | . | . . .....falsecase .....
. | . | . . 12. eq(x,u) = false₂
. | . | . . ⊢  x(∈eq) v  by Analyze -3 THEN Unhide
. | . |
. | . .\
. | . . ⊢  P(x)  by Trivial
. |
. .\
. . 7. ∃x∈v.P(x) ⇒ (∃x:{x:T| x(∈eq) v }. P(x))
. . 8. ∃x∈v.P(x) ⇐ (∃x:{x:T| x(∈eq) v }. P(x))
. . 9. ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. P(x)
. . ⊢  P(u) ∨ ∃x∈v.P(x)  by Analyze -1
. . |
. . 9. x : {x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }
. . 10. P(x)
. . ⊢  P(u) ∨ ∃x∈v.P(x)  by Analyze -2 THEN Unhide
. . |
. . 9. x : T
. . 10. if eq(x,u)→ true₂ else x(∈eq) v fi
. . 11. P(x)
. . ⊢  P(u) ∨ ∃x∈v.P(x)  by Decide1 (x = u)
. . |
. . |\
. . | .....decidable? .....
. . | ⊢  Dec(x = u)  by DiscreteEq 2
. . | |
. . | 3. eq ∈ T→T→𝔹
. . | 4. ∀x,y:T. eq(x,y) ⟺ x = y
. . | 5. P : T→Type
. . | 6. T List
. . | 7. u : T
. . | 8. v : T List
. . | 9. ∃x∈v.P(x) ⇒ (∃x:{x:T| x(∈eq) v }. P(x))
. . | 10. ∃x∈v.P(x) ⇐ (∃x:{x:T| x(∈eq) v }. P(x))
. . | 11. x : T
. . | 12. if eq(x,u)→ true₂ else x(∈eq) v fi
. . | 13. P(x)
. . | ⊢  Dec(x = u)  by Witness4 x THEN With u (Analyze -1)
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. . | |
. . | 4. P : T→Type
. . | 5. T List
. . | 6. u : T
. . | 7. v : T List
. . | 8. ∃x∈v.P(x) ⇒ (∃x:{x:T| x(∈eq) v }. P(x))
. . | 9. ∃x∈v.P(x) ⇐ (∃x:{x:T| x(∈eq) v }. P(x))
. . | 10. x : T
. . | 11. if eq(x,u)→ true₂ else x(∈eq) v fi
. . | 12. P(x)
. . | 13. eq(x,u) ⟺ x = u
. . | ⊢  Dec(x = u)
. . | | by MoveToConcl -1 THEN GenConcl (eq(x,u) = b) THEN BoolInd -2
. . | |    THEN
. . | |    AbReduce 0
. . | |
. . | |\
. . | | .....decidable? .....
. . | | 13. eq(x,u) = true₂
. . | | 14. True ⇒ x = u
. . | | 15. True ⇐ (x = u)
. . | | ⊢  Dec(x = u)  by Choose [1] THEN HypBackchain THEN Trivial
. . | |
. . | .\
. . | . .....decidable? .....
. . | . 13. eq(x,u) = false₂
. . | . 14. False ⇒ x = u
. . | . 15. False ⇐ (x = u)
. . | . ⊢  Dec(x = u)  by Choose [2]
. . | . |
. . | . ⊢  ¬x = u  by UnfoldTopAb 0
. . |
. . |\
. . | 12. x = u
. . | ⊢  P(u) ∨ ∃x∈v.P(x)  by HypSubst -1 -2
. . | |
. . | 11. P(u)

. . | 12. x = u
. . | ⊢  P(u) ∨ ∃x∈v.P(x)  by Choose [1]
. . |
. . .\
. . . 12. ¬x = u
. . . ⊢  P(u) ∨ ∃x∈v.P(x)  by Choose [2]
. . . |
. . . ⊢  ∃x∈v.P(x)  by HypBackchain
. . . |
. . . ⊢  ∃x:{x:T| x(∈eq) v }. P(x)  by Witness x
. . . |
. . . |\
. . . | .....wf .....
. . . | ⊢  x ∈ {x:T| x(∈eq) v }  by AbSetMemEqTypeCD
. . . | |
. . . | |\
. . . | | ⊢  x ∈ T  by Trivial
. . . | |
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. . . | .\
. . . | . .....set predicate .....
. . . | . ⊢  x(∈eq) v  by DiscreteEq 2
. . . | . |
. . . | . 3. eq ∈ T→T→𝔹
. . . | . 4. ∀x,y:T. eq(x,y) ⟺ x = y
. . . | . 5. P : T→Type
. . . | . 6. T List
. . . | . 7. u : T
. . . | . 8. v : T List
. . . | . 9. ∃x∈v.P(x) ⇒ (∃x:{x:T| x(∈eq) v }. P(x))
. . . | . 10. ∃x∈v.P(x) ⇐ (∃x:{x:T| x(∈eq) v }. P(x))
. . . | . 11. x : T
. . . | . 12. if eq(x,u)→ true₂ else x(∈eq) v fi
. . . | . 13. P(x)
. . . | . 14. ¬x = u
. . . | . ⊢  x(∈eq) v  by Witness4 x THEN With u (Analyze -1)
. . . | . |
. . . | . 4. P : T→Type
. . . | . 5. T List
. . . | . 6. u : T
. . . | . 7. v : T List
. . . | . 8. ∃x∈v.P(x) ⇒ (∃x:{x:T| x(∈eq) v }. P(x))
. . . | . 9. ∃x∈v.P(x) ⇐ (∃x:{x:T| x(∈eq) v }. P(x))
. . . | . 10. x : T
. . . | . 11. if eq(x,u)→ true₂ else x(∈eq) v fi
. . . | . 12. P(x)
. . . | . 13. ¬x = u
. . . | . 14. eq(x,u) ⟺ x = u
. . . | . ⊢  x(∈eq) v  by IfThenElseCases 11
. . . | . |
. . . | . |\
. . . | . | .....truecase .....
. . . | . | 11. eq(x,u) = true₂
. . . | . | 12. true₂
. . . | . | 13. P(x)
. . . | . | 14. ¬x = u
. . . | . | 15. eq(x,u) ⟺ x = u
. . . | . | ⊢  x(∈eq) v
. . . | . | | by FwdThru
. . . | . | |    Thm* ∀P:𝔹. P ⟺ P = true₂    {assert_iff_btrue}
. . . | . | |    [11]
. . . | . | |
. . . | . | 16. eq(x,u)
. . . | . | ⊢  x(∈eq) v  by Analyze 14 THEN HypBackchain
. . . | . |
. . . | . .\
. . . | . . .....falsecase .....
. . . | . . 11. eq(x,u) = false₂
. . . | . . 12. x(∈eq) v
. . . | . . 13. P(x)
. . . | . . 14. ¬x = u
. . . | . . 15. eq(x,u) ⟺ x = u
. . . | . . ⊢  x(∈eq) v  by Trivial
. . . |
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. . . .\
. . . . ⊢  P(x)  by Trivial
END
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