```
list_length_wf_nat  THM

⊢  ∀T:Type. |·| ∈ (T List)→ℕ  by Analyze 0
|
1. T : Type
⊢  |·| ∈ (T List)→ℕ
| by Repeat
|    (Unfolds [‘list_length‘;‘letrec‘;‘letrec_body‘;‘letrec_arg‘] 0)
|
⊢  Y(λl,L. Case of L; nil → 0 ; h.t → 1+l(t)) ∈ (T List)→ℕ
| by RWH YUnrollC 0 THEN Reduce 0
|
⊢  (λL.Case of L
|        nil → 0
|        h.t → 1+Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),t))
|  ∈ (T List)→ℕ  by MemberEqCD
|
2. L : T List
⊢  (Case of L
|     nil → 0
|     h.t → 1+Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),t))
|  ∈ ℕ  by ListInd -1 THEN Reduce 0
|
|\
| ⊢  0 ∈ ℕ  by Auto
|
.\
. 3. T
. 4. v : T List
. 5. (Case of v
. |     nil → 0
. |     h.t → 1+Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),t))
. |  ∈ ℕ
. ⊢  1+Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),v) ∈ ℕ
. | by RWH YUnrollC 0 THEN Reduce 0
. |
. |\
. | 6. T
. | 7. t : T List
. | 8. ℤ
. | ⊢  Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),t) ∈ ℤ
. | | by RWH YUnrollC 0 THEN ListInd -2 THEN Reduce 0
. | |
. | |\
. | | ⊢  0 ∈ ℤ  by Auto
. | |
. | .\
. | . 9. T
. | . 10. v1 : T List
. | . 11. (λl,L. Case of L; nil → 0 ; h.t → 1+l(t))
. | . |   (Y(λl,L. Case of L; nil → 0 ; h.t → 1+l(t))
. | . |   ,v1)
. | . |   ∈ ℤ
. | . ⊢  1+Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),v1) ∈ ℤ
. | . | by RWH YUnrollC 0 THEN OnHyps [-1;0] Reduce
```

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. | . |
. | . 11. (Case of v1
. | .        nil → 0
. | .        h.t → 1+Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),t))
. | .     ∈ ℤ
. | . ⊢  1+(Case of v1
. | .         nil → 0
. | .         h.t → 1+Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),t))
. | .    ∈ ℤ  by Auto
. |
. .\
. . .....set predicate .....
. . ⊢  0≤1+(Case of v
. . |         nil → 0
. . |         h.t → 1+Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),t))
. . | by GenConcl
. . |    ((Case of v
. . |        nil → 0
. . |        h.t → 1+Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),t))
. . |     =
. . |     n)
. . |
. . 6. n : ℕ
. . 7. (Case of v
. .       nil → 0
. .       h.t → 1+Y((λl,L. Case of L; nil → 0 ; h.t → 1+l(t)),t))
. .    =
. .    n
. . ⊢  0≤1+n  by Auto
END
```