```
not_list_all_2_implies_exists_not  THM

⊢  ∀eq:{T≡₂}, P:(T→𝔹), L:T List.
|  ¬∀x∈₂L.P(x) ⇒ (∃x:{x:T| x(∈eq) L }. ¬P(x))  by UnivCD
|
1. T : Type
2. eq : {T≡₂}
3. P : T→𝔹
4. L : T List
5. ¬∀x∈₂L.P(x)
⊢  ∃x:{x:T| x(∈eq) L }. ¬P(x)  by ListInd 4 THEN AbReduce 0
|
|\
| 4. T List
| ⊢  ¬True ⇒ (∃x:{x:T| False }. ¬P(x))
|   by Analyze 0 THEN Analyze -1 THEN Trivial
|
.\
. 5. u : T
. 6. v : T List
. 7. ¬∀x∈₂v.P(x) ⇒ (∃x:{x:T| x(∈eq) v }. ¬P(x))
. ⊢  ¬(P(u)∧₂∀x∈₂v.P(x))
. |  ⇒
. |  (∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. ¬P(x))
. | by Analyze 0
. |
. 8. ¬(P(u)∧₂∀x∈₂v.P(x))
. ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. ¬P(x)
. | by RWH (LemmaC Thm* ∀p,q:𝔹. (p∧₂q) ⟺ p & q    {assert_of_band})
. |    -1
. |
. 8. ¬(P(u) & ∀x∈₂v.P(x))
. ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. ¬P(x)
. | by Inst
. |    Thm* Dec(P) ⇒ Dec(Q) ⇒ (¬(P & Q) ⟺ ¬P ∨ ¬Q)    {demorgan2}
. |    [P(u);∀x∈₂v.P(x)]
. |
. 9. ¬(P(u) & ∀x∈₂v.P(x)) ⟺ ¬P(u) ∨ ¬∀x∈₂v.P(x)
. ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. ¬P(x)
. | by Analyze -1 THEN Thin -1 THEN FwdThru -1 [-2]
. |
. 9. ¬(P(u) & ∀x∈₂v.P(x)) ⇒ ¬P(u) ∨ ¬∀x∈₂v.P(x)
. 10. ¬P(u) ∨ ¬∀x∈₂v.P(x)
. ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. ¬P(x)
. | by Analyze -1
. |
. |\
. | 10. ¬P(u)
. | ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. ¬P(x)
. | | by Witness u
. | |
. | |\
. | | .....wf .....
. | | ⊢  u ∈ {x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }  by Analyze
. | | |
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. | | .....set predicate .....
. | | ⊢  if eq(u,u)→ true₂ else u(∈eq) v fi
. | | | by Subst (eq(u,u) = true₂) 0
. | | |
. | | |\
. | | | .....equality .....
. | | | ⊢  eq(u,u) = true₂
. | | |   by BackThru Thm* ∀P:𝔹. P ⟺ P = true₂    {assert_iff_btrue}
. | | |      THEN
. | | |      BackThru
. | | |      Thm* ∀eq:{T≡₂}, u:T. eq(u,u)    {equivalence_reflexive}
. | | |
. | | .\
. | | . ⊢  if true₂→ true₂ else u(∈eq) v fi  by Reduce 0 THEN Trivial
. | |
. | .\
. | . ⊢  ¬P(u)  by Auto
. |
. .\
. . 10. ¬∀x∈₂v.P(x)
. . ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. ¬P(x)
. . | by FwdThru 7 [-1]
. . |
. . 11. ∃x:{x:T| x(∈eq) v }. ¬P(x)
. . ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. ¬P(x)
. . | by Analyze -1
. . |
. . 11. x : {x:T| x(∈eq) v }
. . 12. ¬P(x)
. . ⊢  ∃x:{x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }. ¬P(x)
. . | by Witness x
. . |
. . .....wf .....
. . ⊢  x ∈ {x:T| if eq(x,u)→ true₂ else x(∈eq) v fi }  by Analyze
. . |
. . .....set predicate .....
. . ⊢  if eq(x,u)→ true₂ else x(∈eq) v fi
. . | by Analyze 11 THEN UnhideSqStableHyp 12
. . |
. . 11. x : T
. . 12. x(∈eq) v
. . 13. ¬P(x)
. . ⊢  if eq(x,u)→ true₂ else x(∈eq) v fi  by IfThenElseCases 0
. . |
. . .....truecase .....
. . 14. eq(x,u) = true₂
. . ⊢  true₂  by Reduce 0 THEN Trivial

END
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