```
not_list_all_not_iff_list_exists  THM

⊢  ∀P:(T→Prop).
|  (∀x:T. Dec(P(x))) ⇒ (∀L:T List. ¬∀x∈L.(¬P(x)) ⟺ ∃x∈L.P(x))
| by UnivCD
|
1. T : Type
2. P : T→Prop
3. ∀x:T. Dec(P(x))
4. L : T List
⊢  ¬∀x∈L.(¬P(x)) ⟺ ∃x∈L.P(x)  by Analyze 0 THEN Analyze 0
|
|\
| 5. ¬∀x∈L.(¬P(x))
| ⊢  ∃x∈L.P(x)  by ListInd 4
| |
| |\
| | 4. T List
| | ⊢  ¬∀x∈nil.(¬P(x)) ⇒ ∃x∈nil.P(x)
| |   by AbReduce 0 THEN Analyze -1 THEN Trivial
| |
| .\
| . 5. u : T
| . 6. v : T List
| . 7. ¬∀x∈v.(¬P(x)) ⇒ ∃x∈v.P(x)
| . ⊢  ¬∀x∈(u.v).(¬P(x)) ⇒ ∃x∈(u.v).P(x)
| . | by AbReduce 0 THEN Analyze 0
| . |
| . 8. ¬(¬P(u) & ∀x∈v.(¬P(x)))
| . ⊢  P(u) ∨ ∃x∈v.P(x)
| . | by Inst
| . |    Thm* Dec(P) ⇒ Dec(Q) ⇒ (¬(P & Q) ⟺ ¬P ∨ ¬Q)    {demorgan2}
| . |    [¬P(u);∀x∈v.(¬P(x))]
| . |
| . 9. ¬(¬P(u) & ∀x∈v.(¬P(x))) ⟺ ¬¬P(u) ∨ ¬∀x∈v.(¬P(x))
| . ⊢  P(u) ∨ ∃x∈v.P(x)  by Witness3 u
| . |
| . 3. T List
| . 4. u : T
| . 5. v : T List
| . 6. ¬∀x∈v.(¬P(x)) ⇒ ∃x∈v.P(x)
| . 7. ¬(¬P(u) & ∀x∈v.(¬P(x)))
| . 8. ¬(¬P(u) & ∀x∈v.(¬P(x))) ⟺ ¬¬P(u) ∨ ¬∀x∈v.(¬P(x))
| . 9. Dec(P(u))
| . ⊢  P(u) ∨ ∃x∈v.P(x)  by Unfold ‘decidable‘ -1 THEN Analyze -1
| . |
| . |\
| . | 9. P(u)
| . | ⊢  P(u) ∨ ∃x∈v.P(x)  by Choose [1]
| . |
| . .\
| . . 9. ¬P(u)
| . . ⊢  P(u) ∨ ∃x∈v.P(x)  by Choose [2] THEN HypBackchain
| . . |
| . . ⊢  ¬∀x∈v.(¬P(x))  by Analyze -2 THEN Thin -2
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| . . |
| . . 8. ¬(¬P(u) & ∀x∈v.(¬P(x))) ⇒ ¬¬P(u) ∨ ¬∀x∈v.(¬P(x))
| . . 9. ¬P(u)
| . . ⊢  ¬∀x∈v.(¬P(x))  by FwdThru -2 [-3]
| . . |
| . . 10. ¬¬P(u) ∨ ¬∀x∈v.(¬P(x))
| . . ⊢  ¬∀x∈v.(¬P(x))  by Analyze -1
| . . |
| . . |\
| . . | 10. ¬¬P(u)
| . . | ⊢  ¬∀x∈v.(¬P(x))  by Analyze -1 THEN Trivial
| . . |
| . . .\
| . . . 10. ¬∀x∈v.(¬P(x))
| . . . ⊢  ¬∀x∈v.(¬P(x))  by Trivial
|
.\
. 5. ∃x∈L.P(x)
. ⊢  ¬∀x∈L.(¬P(x))  by ListInd 4
. |
. |\
. | 4. T List
. | ⊢  ∃x∈nil.P(x) ⇒ ¬∀x∈nil.(¬P(x))  by AbReduce 0
. |
. .\
. . 5. u : T
. . 6. v : T List
. . 7. ∃x∈v.P(x) ⇒ ¬∀x∈v.(¬P(x))
. . ⊢  ∃x∈(u.v).P(x) ⇒ ¬∀x∈(u.v).(¬P(x))
. . | by AbReduce 0 THEN Analyze 0
. . |
. . 8. P(u) ∨ ∃x∈v.P(x)
. . ⊢  ¬(¬P(u) & ∀x∈v.(¬P(x)))  by Analyze -1
. . |
. . |\
. . | 8. P(u)
. . | ⊢  ¬(¬P(u) & ∀x∈v.(¬P(x)))  by Analyze 0
. . | |
. . | 9. ¬P(u) & ∀x∈v.(¬P(x))
. . | ⊢  False  by Analyze -1
. . |
. . .\
. . . 8. ∃x∈v.P(x)
. . . ⊢  ¬(¬P(u) & ∀x∈v.(¬P(x)))  by Analyze 0
. . . |
. . . 9. ¬P(u)
. . . 10. ∀x∈v.(¬P(x))

. . . ⊢  False  by FwdThru 7 [8]
END
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