```
remove_is_member_lemma  THM

⊢  ∀eq:{T≡₂}, u:T, L:T List, v:T. v(∈eq) remove(eq;u;L) ⇒ v(∈eq) L
| by Analyze 0 THEN Analyze 0
|
1. T : Type
2. eq : {T≡₂}
⊢  ∀u:T, L:T List, v:T. v(∈eq) remove(eq;u;L) ⇒ v(∈eq) L
| by Equivalence -1
|
3. eq ∈ T→T→𝔹
4. ∀x:T. eq(x,x)
5. ∀x,y:T. eq(x,y) ⟺ eq(y,x)
6. ∀x,y,z:T. eq(x,y) ⇒ eq(y,z) ⇒ eq(x,z)
⊢  ∀u:T, L:T List, v:T. v(∈eq) remove(eq;u;L) ⇒ v(∈eq) L  by UnivCD
|
7. u : T
8. L : T List
9. v : T
10. v(∈eq) remove(eq;u;L)
⊢  v(∈eq) L  by ListInd 8
|
|\
| 9. v : T
| ⊢  v(∈eq) remove(eq;u;nil) ⇒ v(∈eq) nil  by Reduce 0
|
.\
. 10. u1 : T
. 11. v1 : T List
. 12. v(∈eq) remove(eq;u;v1) ⇒ v(∈eq) v1
. ⊢  v(∈eq) remove(eq;u;(u1.v1)) ⇒ v(∈eq) (u1.v1)  by Analyze 0
. |
. 13. v(∈eq) remove(eq;u;(u1.v1))
. ⊢  v(∈eq) (u1.v1)
. | by Rewrite (HigherC remove_unrollC) -1 THEN IfThenElseCases -1
. |
. |\
. | .....truecase .....
. | 13. eq(u,u1) = true₂
. | 14. v(∈eq) v1
. | ⊢  v(∈eq) (u1.v1)
. | | by Rewrite (HigherC is_member_unrollC) 0 THEN IfThenElseCases 0
. | |
. | |\
. | | .....truecase .....
. | | 15. eq(v,u1) = true₂
. | | ⊢  true₂  by Rewrite assert_evalC 0 THEN Trivial
. | |
. | .\
. | . .....falsecase .....
. | . 15. eq(v,u1) = false₂
. | . ⊢  v(∈eq) v1  by Trivial
. |
. .\
. . .....falsecase .....
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. . 13. eq(u,u1) = false₂
. . 14. v(∈eq) (u1.remove(eq;u;v1))
. . ⊢  v(∈eq) (u1.v1)
. . | by Rewrite (HigherC is_member_unrollC) 0 THEN IfThenElseCases 0
. . |
. . |\
. . | .....truecase .....
. . | 15. eq(v,u1) = true₂
. . | ⊢  true₂  by Rewrite assert_evalC 0 THEN Trivial
. . |
. . .\
. . . .....falsecase .....
. . . 15. eq(v,u1) = false₂
. . . ⊢  v(∈eq) v1  by HypBackchain
. . . |
. . . ⊢  v(∈eq) remove(eq;u;v1)
. . . | by Rewrite (HigherC is_member_unrollC) 14
. . . |    THEN
. . . |    IfThenElseCases 14
. . . |
. . . |\
. . . | .....truecase .....
. . . | 14. eq(v,u1) = true₂
. . . | 15. true₂
. . . | 16. eq(v,u1) = false₂
. . . | ⊢  v(∈eq) remove(eq;u;v1)  by HypSubst 14 16
. . . | |
. . . | 16. true₂ = false₂
. . . | ⊢  v(∈eq) remove(eq;u;v1)
. . . |   by ApFunToHypEquands ‘b’ if b→ 0 else 1 fi ℤ -1
. . . |      THEN
. . . |      Rewrite (HigherC ifthenelse_evalC) -1
. . . |
. . . .\
. . . . .....falsecase .....
. . . . 14. eq(v,u1) = false₂
. . . . 15. v(∈eq) remove(eq;u;v1)
. . . . 16. eq(v,u1) = false₂
. . . . ⊢  v(∈eq) remove(eq;u;v1)  by Trivial
END
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