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sublist_functionality_wrt_id_id_list_iso  THM

⊢  ∀L1,L2:T List, eq1,eq2:{T≡₂}, L3,L4:T List.
|  Discrete{T}
|  ⇒
|  eq1 = eq2 ⇒ L1 = L2 ⇒ L3(∼eq1)L4 ⇒ (L1(⊆eq2)L3 ⟺ L2(⊆eq2)L4)
| by Analyze 0 THEN Analyze 0 THEN Analyze 0 THEN Analyze 0
|    THEN
|    Equivalence -1
|    THEN
|    Analyze 0
|    THEN
|    Equivalence -1
|    THEN
|    UnivCD
|
1. T : Type
2. L1 : T List
3. L2 : T List
4. eq1 : {T≡₂}
5. eq1 ∈ T→T→𝔹
6. ∀x:T. eq1(x,x)
7. ∀x,y:T. eq1(x,y) ⟺ eq1(y,x)
8. ∀x,y,z:T. eq1(x,y) ⇒ eq1(y,z) ⇒ eq1(x,z)
9. eq2 : {T≡₂}
10. eq2 ∈ T→T→𝔹
11. ∀x:T. eq2(x,x)
12. ∀x,y:T. eq2(x,y) ⟺ eq2(y,x)
13. ∀x,y,z:T. eq2(x,y) ⇒ eq2(y,z) ⇒ eq2(x,z)
14. L3 : T List
15. L4 : T List
16. Discrete{T}
17. eq1 = eq2
18. L1 = L2
19. L3(∼eq1)L4
⊢  L1(⊆eq2)L3 ⟺ L2(⊆eq2)L4
| by RevHypSubst 17 0 THENA Equivalence -1 THEN RenameVar ‘eq’ 4
|    THEN
|    OnHyps [17;13;12;11;10;9] (λi.Thin i)
|
4. eq : {T≡₂}
5. eq ∈ T→T→𝔹
6. ∀x:T. eq(x,x)
7. ∀x,y:T. eq(x,y) ⟺ eq(y,x)
8. ∀x,y,z:T. eq(x,y) ⇒ eq(y,z) ⇒ eq(x,z)
9. L3 : T List
10. L4 : T List
11. Discrete{T}
12. L1 = L2
13. L3(∼eq)L4
⊢  L1(⊆eq)L3 ⟺ L2(⊆eq)L4
| by UnfoldTopAb -1 THEN UnfoldTopAb -1 THEN Analyze -1
|    THEN
|    HypSubst 12 0
|    THEN
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|    Analyze 0
|    THEN
|    Analyze 0
|
|\
| 13. L3(⊆eq)L4
| 14. L4(⊆eq)L3
| 15. L2(⊆eq)L3
| ⊢  L2(⊆eq)L4
| | by Inst
| |    Thm* Discrete{T}
| |         ⇒
| |         (∀eq1,eq2,eq3:{T≡₂}, L1,L2,L3:T List.
| |          eq1 = eq2
| |          ⇒
| |          eq2 = eq3 ⇒ L1(⊆eq1)L2 ⇒ L2(⊆eq2)L3 ⇒ L1(⊆eq3)L3)
| |    {sublist_transitivity}
| |    [T;eq;eq;eq;L2;L1;L3]
| |    THEN
| |    Inst
| |    Thm* Discrete{T}
| |         ⇒
| |         (∀eq1,eq2,eq3:{T≡₂}, L1,L2,L3:T List.
| |          eq1 = eq2
| |          ⇒
| |          eq2 = eq3 ⇒ L1(⊆eq1)L2 ⇒ L2(⊆eq2)L3 ⇒ L1(⊆eq3)L3)
| |    {sublist_transitivity}
| |    [T;eq;eq;eq;L2;L3;L4]
| |
| |\
| | 16. L2(⊆eq)L4
| | ⊢  L2(⊆eq)L1  by HypSubst 12 0
| | |
| | ⊢  L2(⊆eq)L2
| |   by BackThru
| |      Thm* Discrete{T} ⇒ (∀eq:{T≡₂}, L,M:T List. L = M ⇒ L(⊆eq)M)
| |      {sublist_weakening_wrt_identity}
| |
| |\
| | 16. L2(⊆eq)L4
| | ⊢  L1(⊆eq)L3  by RevHypSubst 12 15 THEN Trivial
| |
| .\
| . 16. L2(⊆eq)L3
| . 17. L2(⊆eq)L4
| . ⊢  L2(⊆eq)L4  by Auto
|

.\
. 13. L3(⊆eq)L4
. 14. L4(⊆eq)L3
. 15. L2(⊆eq)L4
. ⊢  L2(⊆eq)L3
. | by Inst
. |    Thm* Discrete{T}
. |         ⇒
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. |          (∀eq1,eq2,eq3:{T≡₂}, L1,L2,L3:T List.
. |           eq1 = eq2
. |           ⇒
. |           eq2 = eq3 ⇒ L1(⊆eq1)L2 ⇒ L2(⊆eq2)L3 ⇒ L1(⊆eq3)L3)
. |     {sublist_transitivity}
. |     [T;eq;eq;eq;L1;L2;L4]
. |     THEN
. |     Inst
. |     Thm* Discrete{T}
. |          ⇒
. |          (∀eq1,eq2,eq3:{T≡₂}, L1,L2,L3:T List.
. |           eq1 = eq2
. |           ⇒
. |           eq2 = eq3 ⇒ L1(⊆eq1)L2 ⇒ L2(⊆eq2)L3 ⇒ L1(⊆eq3)L3)
. |     {sublist_transitivity}
. |     [T;eq;eq;eq;L1;L4;L3]
. |
. |\
. | .....antecedent .....
. | ⊢  L1(⊆eq)L4  by RevHypSubst 12 15 THEN Trivial
. |
. |\
. | 16. L1(⊆eq)L3
. | ⊢  L1(⊆eq)L2
. |   by HypSubst 12 0
. |      THEN
. |      BackThru
. |      Thm* Discrete{T} ⇒ (∀eq:{T≡₂}, L,M:T List. L = M ⇒ L(⊆eq)M)
. |      {sublist_weakening_wrt_identity}
. |
. .\
. . 16. L1(⊆eq)L4
. . 17. L1(⊆eq)L3
. . ⊢  L2(⊆eq)L3  by HypSubst 12 17 THEN Trivial
END
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