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apply_alist_member  THM

⊢  ∀T:Type, as:(Label×T) List, d:T, x:Label.
|  (x ∈ 1of(unzip(as))) ⇒ (<x,apply_alist(as;x;d)> ∈ as)
| by D 0 THENM D 0 THENM ListInd (-1) THENA Auto
...
|
|\
| 1. T : Type
| 2. (Label×T) List
| ⊢  ∀d:T, x:Label. (x ∈ nil) ⇒ (<x,apply_alist(nil;x;d)> ∈ nil)
|   by Unfold `l_member` 0 THEN Reduce 0 THEN Auto
| ...
|
.\
. 1. T : Type
. 2. (Label×T) List
. 3. u : Label×T
. 4. v : (Label×T) List
. 5. ∀d:T, x:Label.
. |  (x ∈ map(λp.1of(p);v)) ⇒ (<x,apply_alist(v;x;d)> ∈ v)
. ⊢  ∀d:T, x:Label.
. |  (x ∈ [1of(u) / map(λp.1of(p);v)]) ⇒
. |  (<x,apply_alist([u / v];x;d)> ∈ [u / v])
. | by RWW "cons_member" 0
. THEN Auto
. |
. 1. T : Type
. 2. (Label×T) List
. 3. u : Label×T
. 4. v : (Label×T) List
. 5. ∀d:T, x:Label.
. |  (x ∈ map(λp.1of(p);v)) ⇒ (<x,apply_alist(v;x;d)> ∈ v)
. 6. d : T
. 7. x : Label
. 8. x = 1of(u) ∨ (x ∈ map(λp.1of(p);v))
. ⊢  <x,apply_alist([u / v];x;d)> = u
. |  ∨ (<x,apply_alist([u / v];x;d)> ∈ v)
. | by InstLemma `apply_alist_cons` [T;v;u;x;d]
. ...
. THENA Auto
. |
. 1. T : Type
. 2. (Label×T) List
. 3. u : Label×T
. 4. v : (Label×T) List
. 5. ∀d:T, x:Label.
. |  (x ∈ map(λp.1of(p);v)) ⇒ (<x,apply_alist(v;x;d)> ∈ v)
. 6. d : T
. 7. x : Label
. 8. x = 1of(u) ∨ (x ∈ map(λp.1of(p);v))
. 9. apply_alist([u / v];x;d) ~ if 1of(u) =₁ x→ 2of(u)
. |                              else apply_alist(v;x;d) fi
. ⊢  <x,if 1of(u) =₁ x→ 2of(u) else apply_alist(v;x;d) fi> = u
. |  ∨ (<x,if 1of(u) =₁ x→ 2of(u) else apply_alist(v;x;d) fi> ∈ v)
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. | by SplitITE 0 THENA Auto THEN Try TrivialOr
. |
. |\
. | .....truecase .....
. | 1. T : Type
. | 2. (Label×T) List
. | 3. u : Label×T
. | 4. v : (Label×T) List
. | 5. ∀d:T, x:Label.
. | |  (x ∈ map(λp.1of(p);v)) ⇒ (<x,apply_alist(v;x;d)> ∈ v)
. | 6. d : T
. | 7. x : Label
. | 8. x = 1of(u) ∨ (x ∈ map(λp.1of(p);v))
. | 9. apply_alist([u / v];x;d) ∼ if 1of(u) =₁ x→ 2of(u)
. | |                              else apply_alist(v;x;d) fi
. | 10. 1of(u) = x
. | ⊢  <x,2of(u)> = u ∨ (<x,2of(u)> ∈ v)  by Sel 1 (D 0) THENA Auto
. | |
. | 1. T : Type
. | 2. (Label×T) List
. | 3. u : Label×T
. | 4. v : (Label×T) List
. | 5. ∀d:T, x:Label.
. | |  (x ∈ map(λp.1of(p);v)) ⇒ (<x,apply_alist(v;x;d)> ∈ v)
. | 6. d : T
. | 7. x : Label
. | 8. x = 1of(u) ∨ (x ∈ map(λp.1of(p);v))
. | 9. apply_alist([u / v];x;d) ∼ if 1of(u) =₁ x→ 2of(u)
. | |                              else apply_alist(v;x;d) fi
. | 10. 1of(u) = x
. | ⊢  <x,2of(u)> = u  by D 3 THEN All Reduce THEN EqCD THEN Auto
. | |
. | .....subterm .....
. | 1. T : Type
. | 2. (Label×T) List
. | 3. u1 : Label
. | 4. u2 : T
. | 5. v : (Label×T) List
. | 6. ∀d:T, x:Label.
. |    (x ∈ map(λp.1of(p);v)) ⇒ (<x,apply_alist(v;x;d)> ∈ v)
. | 7. d : T
. | 8. x : Label
. | 9. x = u1 ∨ (x ∈ map(λp.1of(p);v))
. | 10. apply_alist([<u1,u2> / v];x;d) ∼ if u1 =₁ x→ u2
. |                                      else apply_alist(v;x;d) fi
. | 11. u1 = x
. | ⊢  x = u1  by D 8 THEN D 3 THEN Subst u1 ∼ x 0 THEN Auto

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. . .....falsecase .....
. . 1. T : Type
. . 2. (Label×T) List
. . 3. u : Label×T
. . 4. v : (Label×T) List
. . 5. ∀d:T, x:Label.
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. . |  (x ∈ map(λp.1of(p);v)) ⇒ (<x,apply_alist(v;x;d)> ∈ v)
. . 6. d : T
. . 7. x : Label
. . 8. x = 1of(u) ∨ (x ∈ map(λp.1of(p);v))
. . 9. apply_alist([u / v];x;d) ~ if 1of(u) =1 x→ 2of(u)
. . |                              else apply_alist(v;x;d) fi
. . 10. ¬1of(u) = x
. . ⊢  <x,apply_alist(v;x;d)> = u ∨ (<x,apply_alist(v;x;d)> ∈ v)
. . | by D (-3)
. . |
. . |\
. . | 1. T : Type
. . | 2. (Label×T) List
. . | 3. u : Label×T
. . | 4. v : (Label×T) List
. . | 5. ∀d:T, x:Label.
. . |    (x ∈ map(λp.1of(p);v)) ⇒ (<x,apply_alist(v;x;d)> ∈ v)
. . | 6. d : T
. . | 7. x : Label
. . | 8. x = 1of(u)
. . | 9. apply_alist([u / v];x;d) ~ if 1of(u) =1 x→ 2of(u)
. . |                                else apply_alist(v;x;d) fi
. . | 10. ¬1of(u) = x
. . | ⊢  <x,apply_alist(v;x;d)> = u ∨ (<x,apply_alist(v;x;d)> ∈ v)
. . |   by D (-1) THEN Subst x ~ 1of(u) 0 THEN Auto
. . |
. . .\
. . . 1. T : Type
. . . 2. (Label×T) List
. . . 3. u : Label×T
. . . 4. v : (Label×T) List
. . . 5. ∀d:T, x:Label.
. . . |  (x ∈ map(λp.1of(p);v)) ⇒ (<x,apply_alist(v;x;d)> ∈ v)
. . . 6. d : T
. . . 7. x : Label
. . . 8. (x ∈ map(λp.1of(p);v))
. . . 9. apply_alist([u / v];x;d) ~ if 1of(u) =1 x→ 2of(u)
. . . |                              else apply_alist(v;x;d) fi
. . . 10. ¬1of(u) = x
. . . ⊢  <x,apply_alist(v;x;d)> = u ∨ (<x,apply_alist(v;x;d)> ∈ v)
. . . | by Sel 2 (D 0) THENA Auto
. . . |
. . . 1. T : Type
. . . 2. (Label×T) List
. . . 3. u : Label×T
. . . 4. v : (Label×T) List
. . . 5. ∀d:T, x:Label.
. . .    (x ∈ map(λp.1of(p);v)) ⇒ (<x,apply_alist(v;x;d)> ∈ v)
. . . 6. d : T
. . . 7. x : Label
. . . 8. (x ∈ map(λp.1of(p);v))
. . . 9. apply_alist([u / v];x;d) ~ if 1of(u) =1 x→ 2of(u)
. . .                              else apply_alist(v;x;d) fi
. . . 10. ¬1of(u) = x
. . . ⊢  (<x,apply_alist(v;x;d)> ∈ v)  by BackThruSomeHyp THEN Auto
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END
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