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apply_alist_property  THM

⊢  ∀T:Type, as:(Label×T) List, d:T, x:Label.
|  (apply_alist(as;x;d) ∈ 2of(unzip(as))) ∨ apply_alist(as;x;d) = d
| by Auto THEN Unfold ‘apply_alist‘ 0

|    THEN InstLemma ‘find_property‘ [Label×T;λp.1of(p) =₁ x;as;<x,d>
|    ] THENA Auto
THEN ParallelOp (-1)
|
|\
| 1. T : Type
| 2. as : (Label×T) List
| 3. d : T
| 4. x : Label
| 5. ((first a ∈ as s.t. (λp.1of(p) =₁ x)(a) else <x,d>) ∈ as)
| ⊢  (2of((first p ∈ as s.t. 1of(p) =₁ x else <x,d>)) ∈ ...)
| | by RepeatFor 2 (ParallelOp (-1))
| |
| 1. T : Type
| 2. as : (Label×T) List
| 3. d : T
| 4. x : Label
| 5. i : ℕ
| 6. i<||as||
| |  & (first a ∈ as s.t. (λp.1of(p) =₁ x)(a) else <x,d>) = as[i]
| ⊢  i<||2of(unzip(as))||
| |  & 2of((first p ∈ as s.t. 1of(p) =₁ x else <x,d>))
| |    =
| |    2of(unzip(as))[i]
| | by Unfold ‘unzip‘ 0 THEN Reduce 0 THEN ExRepD THEN Auto
| |
| |\
| | 1. T : Type
| | 2. as : (Label×T) List
| | 3. d : T
| | 4. x : Label
| | 5. i : ℕ
| | 6. i<||as||
| | 7. (first a ∈ as s.t. (λp.1of(p) =₁ x)(a) else <x,d>) = as[i]
| | ⊢  i<||map(λp.2of(p);as)||  by RWO "map_length" 0 THEN Auto
| |
| .\
| . 1. T : Type
| . 2. as : (Label×T) List
| . 3. d : T
| . 4. x : Label
| . 5. i : ℕ
| . 6. i<||as||
| . 7. (first a ∈ as s.t. (λp.1of(p) =₁ x)(a) else <x,d>) = as[i]
| . ⊢  2of((first p ∈ as s.t. 1of(p) =₁ x else <x,d>))
| . |  =
| . |  map(λp.2of(p);as)[i]
| . | by ...
| . |
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| . |\
| . | 1. T : Type
| . | 2. as : (Label×T) List
| . | 3. d : T
| . | 4. x : Label
| . | 5. i : ℕ
| . | 6. i<||as||
| . | 7. (first a ∈ as s.t. 1of(a) =₁ x else <x,d>) = as[i]
| . | ⊢  2of(as[i]) = map(λp.2of(p);as)[i]
| . | | by RW0 "map_select" 0 THEN Auto
| . | |
| . | 1. T : Type
| . | 2. as : (Label×T) List
| . | 3. d : T
| . | 4. x : Label
| . | 5. i : ℕ
| . | 6. i<||as||
| . | 7. (first a ∈ as s.t. 1of(a) =₁ x else <x,d>) = as[i]
| . | ⊢  2of(as[i]) = (λp.2of(p))(as[i])  by Reduce 0 THEN Auto
| . |
| . .\
| . . 1. T : Type
| . . 2. as : (Label×T) List
| . . 3. d : T
| . . 4. x : Label
| . . 5. i : ℕ
| . . 6. i<||as||
| . . 7. (first a ∈ as s.t. 1of(a) =₁ x else <x,d>) = as[i]
| . . 8. Label×T
| . . ⊢  i<||map(λp.2of(p);as)||  by RW0 "map_length" 0 THEN Auto
|
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. 1. T : Type
. 2. as : (Label×T) List
. 3. d : T
. 4. x : Label
. 5. (first a ∈ as s.t. (λp.1of(p) =₁ x)(a) else <x,d>) = <x,d>
. ⊢  2of((first p ∈ as s.t. 1of(p) =₁ x else <x,d>)) = d
.   by ...
END
```