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member_rel_vars  THM

⊢  ∀x:Label, r:rel().
|  (x ∈ rel_vars(r))
|  ⟺
|  (∃i:ℕ. i<||r.args|| & (x ∈ term_vars(r.args[i])))
| by Unfold ‘rel_vars‘ 0 THEN Auto THEN D 2 THEN All Reduce

|    THEN ListInd (-2) THEN Reduce 0
THEN Auto

|    THEN ExRepD THEN Auto
|
|\
| 1. x : Label
| 2. relname()
| 3. Term List
| 4. (x ∈ nil)
| ⊢  ∃i:ℕ. i<0 & (x ∈ term_vars(nil[i]))
|   by Unfold ‘l_member‘ (-1) THEN ExRepD THEN All Reduce THEN Auto
|
|\
| 1. x : Label
| 2. relname()
| 3. Term List
| 4. u : Term
| 5. v : Term List
| 6. (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| |  ⇒
| |  (∃i:ℕ. i<||v|| & (x ∈ term_vars(v[i])))
| 7. (x ∈ term_vars(u) @ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| ⊢  ∃i:ℕ. i<||v||+1 & (x ∈ term_vars([u / v][i]))
| | by ...
| |
| |\
| | 1. x : Label
| | 2. relname()
| | 3. Term List
| | 4. u : Term
| | 5. v : Term List
| | 6. (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| | |  ⇒
| | |  (∃i:ℕ. i<||v|| & (x ∈ term_vars(v[i])))
| | 7. (x ∈ term_vars(u))
| | ⊢  ∃i:ℕ. i<||v||+1 & (x ∈ term_vars([u / v][i]))
| | | by InstConcl [0] THENA (Auto THEN Reduce 0 THEN Auto)
| | |
| | |    THEN D 0 THEN Auto'
| | |
| | 1. x : Label
| | 2. relname()
| | 3. Term List
| | 4. u : Term
| | 5. v : Term List
| | 6. (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
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| |    ⇒
| |    (∃i:ℕ. i<||v|| & (x ∈ term_vars(v[i])))
| | 7. (x ∈ term_vars(u))
| | 8. 0<||v||+1
| | ⊢  (x ∈ term_vars([u / v][0]))
| |   by RWW "select_cons_hd" 0 THEN Auto
| |
| .\
| . 1. x : Label
| . 2. relname()
| . 3. Term List
| . 4. u : Term
| . 5. v : Term List
| . 6. (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| . 7. ∃i:ℕ. i<||v|| & (x ∈ term_vars(v[i]))
| . ⊢  ∃i:ℕ. i<||v||+1 & (x ∈ term_vars([u / v][i]))
| . | by ExRepD THEN InstConcl [i+1
| . |    ] THENA (Auto THEN Reduce 0 THEN Auto)
| . |
| . 1. x : Label
| . 2. relname()
| . 3. Term List
| . 4. u : Term
| . 5. v : Term List
| . 6. (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| . 7. i : ℕ
| . 8. i<||v||
| . 9. (x ∈ term_vars(v[i]))
| . ⊢  i+1<||v||+1 & (x ∈ term_vars([u / v][(i+1)]))
| .   by D 0 THEN Auto' THEN RWW "select_cons_tl" 0
| .
| .      THENM ArithSimp 0 THEN  Auto
|
|\
| 1. x : Label
| 2. relname()
| 3. Term List
| 4. u : Term
| 5. v : Term List
| 6. (∃i:ℕ. i<||v|| & (x ∈ term_vars(v[i])))
| |  ⇒
| |  (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| 7. i : ℕ
| 8. i<||v||+1
| 9. (x ∈ term_vars([u / v][i]))
| ⊢  (x ∈ term_vars(u)) @ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| | by MoveToConcl (-1) THEN CaseNat 0 'i' THEN Reduce 0
|
| |    THEN RWW "member_append" 0 THEN Auto THEN Reduce 0
|
| |    THEN Auto THEN TrivialOr
| |
| 1. x : Label
| 2. relname()
| 3. Term List
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| 4. u : Term
| 5. v : Term List
| 6. (∃i:ℕ. i<||v|| & (x ∈ term_vars(v[i])))
| |  ⇒
| |  (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| 7. i : ℕ
| 8. i<||v||+1
| 9. ¬i = 0
| 10. (x ∈ term_vars([u / v][i]))
| ⊢  (x ∈ term_vars(u)) ∨ (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| | by Sel 2 (D 0) THENA Auto THEN BackThruSomeHyp
| |
| 1. x : Label
| 2. relname()
| 3. Term List
| 4. u : Term
| 5. v : Term List
| 6. (∃i:ℕ. i<||v|| & (x ∈ term_vars(v[i])))
| |  ⇒
| |  (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| 7. i : ℕ
| 8. i<||v||+1
| 9. ¬i = 0
| 10. (x ∈ term_vars([u / v][i]))
| ⊢  ∃i:ℕ. i<||v|| & (x ∈ term_vars(v[i]))
| | by InstConcl [i-1] THEN Auto
| |
| 1. x : Label
| 2. relname()
| 3. Term List
| 4. u : Term
| 5. v : Term List
| 6. (∃i:ℕ. i<||v|| & (x ∈ term_vars(v[i])))
|    ⇒
|    (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
| 7. i : ℕ
| 8. i<||v||+1
| 9. ¬i = 0
| 10. (x ∈ term_vars([u / v][i]))
| ⊢  (x ∈ term_vars(v[(i-1)]))
|   by RWW "select_cons_tl" (-1) THEN Auto
|
.\
. 1. x : Label
. 2. relname()
. 3. Term List
. 4. u : Term
. 5. v : Term List
. 6. (∃i:ℕ. i<||v|| & (x ∈ term_vars(v[i])))
.    ⇒
.    (x ∈ reduce(λt,vs. term_vars(t) @ vs;nil;v))
. 7. i : ℕ
. 8. i<||v||+1
. ⊢  i<||[u / v]||  by Reduce 0 THEN Auto
END
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