```
member_st_app1  THM

⊢  ∀s1,s2,s:SimpleType. s ∈ st_app1(s1;s2) ⟺ st_eq(s1;s2→s)
| by UnivCD THENA Auto THEN Unfold `st` 1

|    THEN TreeInd 1 THEN Unfolds ``st_app1 arrow`` 0
THEN Reduce 0
|
|\
| 1. Tree(Label+Unit)
| 2. s2 : SimpleType
| 3. s : SimpleType
| 4. u : Tree(Label+Unit)→Type
| 5. s1:{v:Tree(Label+Unit)| u(v) }→s ∈ st_app1(s1;s2)
| |                                   ⟺
| |                                   st_eq(s1;s2→s)
| 6. x : Label+Unit
| ⊢  s ∈ <> ⟺ st_eq(tree_leaf(x);tree_node(<s2, s>))
| | by ...
| |
| 1. Tree(Label+Unit)
| 2. s2 : SimpleType
| 3. s : SimpleType
| 4. u : Tree(Label+Unit)→Type
| 5. s1:{v:Tree(Label+Unit)| u(v) }→s ∈ st_app1(s1;s2)
|                                     ⟺
|                                     st_eq(s1;s2→s)
| 6. x : Label+Unit
| ⊢  False ⟺ st_eq(tree_leaf(x);tree_node(<s2, s>))
|   by RecUnfold `st_eq` 0 THEN Reduce 0 THEN Auto
|
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. 1. Tree(Label+Unit)
. 2. s2 : SimpleType
. 3. s : SimpleType
. 4. u : Tree(Label+Unit)→Type
. 5. s1:{v:Tree(Label+Unit)| u(v) }→s ∈ st_app1(s1;s2)
. |                                   ⟺
. |                                   st_eq(s1;s2→s)
. 6. y1 : {v:Tree(Label+Unit)| u(v) }
. 7. y2 : {v:Tree(Label+Unit)| u(v) }
. ⊢  s ∈ if st_eq(y1;s2)→ <y2> else <> fi
. |  ⟺
. |  st_eq(tree_node(<y1, y2>);tree_node(<s2, s>))
. | by All (Fold `st`) THEN SplitOnConclITE THENA Auto
. |
. |\
. | .....truecase .....
. | 1. SimpleType
. | 2. s2 : SimpleType
. | 3. s : SimpleType
. | 4. u : SimpleType→Type
. | 5. s1:{v:SimpleType| u(v) }→s ∈ st_app1(s1;s2) ⟺ st_eq(s1;s2→s)
. | 6. y1 : {v:SimpleType| u(v) }
. | 7. y2 : {v:SimpleType| u(v) }
```

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. | 8. st_eq(y1;s2)
. | ⊢  s ∈ <y2> ⟺ st_eq(tree_node(<y1, y2>);tree_node(<s2, s>))
. |   by RecUnfold ‘st_eq‘ 0 THEN Reduce 0
. | THEN MoveToConcl (-1)
. |
. |      THEN RW ColMemberC 0 THENM
. | RW assert_pushdownC 0 THENM
. |
. |      RWW "assert_st_eq" 0 THEN Auto
. |
. .\
. . .....falsecase .....
. . 1. SimpleType
. . 2. s2 : SimpleType
. . 3. s : SimpleType
. . 4. u : SimpleType→Type
. . 5. s1:{v:SimpleType| u(v) }→s ∈ st_app1(s1;s2) ⟺ st_eq(s1;s2→s)
. . 6. y1 : {v:SimpleType| u(v) }
. . 7. y2 : {v:SimpleType| u(v) }
. . 8. ¬st_eq(y1;s2)
. . ⊢  s ∈ <> ⟺ st_eq(tree_node(<y1, y2>);tree_node(<s2, s>))
. .   by RecUnfold ‘st_eq‘ 0 THEN Reduce 0
. .
. .      THEN RWW "member_col_none" 0 THENM
. .
. .      RW assert_pushdownC 0 THEN Auto
END
```