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assert_subst_mentions_trace  THM

⊢  ∀as:(Label×Term) List.
|  subst_mentions_trace(as)
|  ⟺
|  (∃i:ℕ||as||. mentions_trace(2of(as[i])))
| by InductionOnList THEN Reduce 0
|
|\
| 1. (Label×Term) List
| ⊢  subst_mentions_trace(nil) ⟺ (∃i:ℕ0. mentions_trace(2of(nil[i])))
|   by Unfold ‘subst_mentions_trace‘ 0 THEN Reduce 0 THEN Auto
|
|      THEN ExRepD THEN Auto’
|
.\
. 1. (Label×Term) List
. 2. u : Label×Term
. 3. v : (Label×Term) List
. 4. subst_mentions_trace(v) ⟺ (∃i:ℕ||v||. mentions_trace(2of(v[i])))
. ⊢  subst_mentions_trace([u / v])
. |  ⟺
. |  (∃i:ℕ(||v||+1). mentions_trace(2of([u / v][i])))
. | by Unfold ‘subst_mentions_trace‘ 0
.
. |    THEN Reduce 0 THEN Fold ‘subst_mentions_trace‘ 0
.
. |    THEN RW assert_pushdownC 0 THENA Auto’
. |
. 1. (Label×Term) List
. 2. u : Label×Term
. 3. v : (Label×Term) List
. 4. subst_mentions_trace(v) ⟺ (∃i:ℕ||v||. mentions_trace(2of(v[i])))
. ⊢  mentions_trace(2of(u)) ∨ subst_mentions_trace(v)
. |  ⟺
. |  (∃i:ℕ(||v||+1). mentions_trace(2of([u / v][i])))
. | by Auto THEN D (-1) THEN AllHyps SimpHyp
. THEN Try (InstConcl [0
. |    ] THEN Reduce 0 THEN Complete Auto’)
.
. |    THEN Try (ExRepD THEN InstConcl [i+1] THEN Auto’
.
. |              THEN RW SelectConsC 0 THEN Auto)
. |
. 1. True
. 2. u : Label×Term
. 3. v : (Label×Term) List
. 4. subst_mentions_trace(v) ⇒ (∃i:ℕ||v||. mentions_trace(2of(v[i])))
. 5. subst_mentions_trace(v) ⇐ (∃i:ℕ||v||. mentions_trace(2of(v[i])))
. 6. i : ℕ(||v||+1)
. 7. mentions_trace(2of([u / v][i]))
. ⊢  mentions_trace(2of(u)) ∨ subst_mentions_trace(v)
. | by CaseNat 0 ‘i’ THENL
.
. |    [OrLeft THENM RWO "select_cons_hd" (-2) THEN Auto;
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.
. |    OrRight THENM RWO "select_cons_tl" (-2) THENA Auto']
. |
. 1. True
. 2. Label×Term
. 3. v : (Label×Term) List
. 4. subst_mentions_trace(v) ⇒ (∃i:ℕ||v||. mentions_trace(2of(v[i])))
. 5. subst_mentions_trace(v) ⇐ (∃i:ℕ||v||. mentions_trace(2of(v[i])))
. 6. i : ℕ(||v||+1)
. 7. mentions_trace(2of(v[(i-1)]))
. 8. ¬i = 0
. ⊢  subst_mentions_trace(v)
.   by BackThruSomeHyp THEN AutoInstConcl [] THEN Auto'
END
```