```
closed_rel_args  THM

⊢  ∀r:rel(), i:ℕ.
|  closed_rel(r) ⇒ i<||r.args|| ⇒ closed_term(r.args[i])
| by Unfolds ``closed_term closed_rel`` 0 THEN

|    Unfold `rel_free_vars` 0 THEN Auto

|    THEN RepeatFor 2 (MoveToConcl (-1))
THEN GenConcl r.args = l

|    THENA Auto
|
1. r : rel()
2. i : ℕ
3. l : Term List
4. r.args = l
⊢  reduce(λt,vs. term_free_vars(t) @ vs;nil;l) = nil ⇒
|  i<||l|| ⇒ term_free_vars(l[i]) = nil  by MoveToConcl 2
|
1. r : rel()
2. l : Term List
3. r.args = l
⊢  ∀i:ℕ.
|  reduce(λt,vs. term_free_vars(t) @ vs;nil;l) = nil ⇒
|  i<||l|| ⇒ term_free_vars(l[i]) = nil
| by Thin (-1) THEN ListInd (-1) THEN Reduce 0
|
|\
| 1. rel()
| 2. Term List
| ⊢  ∀i:ℕ. nil = nil ⇒ i<0 ⇒ term_free_vars(nil[i]) = nil  by Auto
|
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. 1. rel()
. 2. Term List
. 3. u : Term
. 4. v : Term List
. 5. ∀i:ℕ.
. |  reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
. |  i<||v|| ⇒ term_free_vars(v[i]) = nil
. ⊢  ∀i:ℕ.
. |  (term_free_vars(u) @ reduce(λt,vs. term_free_vars(t) @ vs;nil;v))
. |  =
. |  nil
. |  ⇒
. |  i<||v||+1 ⇒ term_free_vars([u / v][i]) = nil
. | by RWW "append_is_nil" 0 THENA (Auto THEN Reduce 0 THEN Auto')
. |
. 1. rel()
. 2. Term List
. 3. u : Term
. 4. v : Term List
. 5. ∀i:ℕ.
. |  reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
```

```
. |  i<||v|| ⇒ term_free_vars(v[i]) = nil
. ⊢  ∀i:ℕ.
. |  term_free_vars(u) = nil
. |  & reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil
. |  ⇒
. |  i<||v||+1 ⇒ term_free_vars([u / v][i]) = nil
. | by Auto THEN CaseNat 0 ‘i’
. |
. |\
. | 1. rel()
. | 2. Term List
. | 3. u : Term
. | 4. v : Term List
. | 5. ∀i:ℕ.
. |    reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
. |    i<||v|| ⇒ term_free_vars(v[i]) = nil
. | 6. i : ℕ
. | 7. term_free_vars(u) = nil
. | 8. reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil
. | 9. i<||v||+1
. | 10. i = 0
. | ⊢  term_free_vars([u / v][0]) = nil
. |   by RWW "select_cons_hd" 0 THEN Auto
. |
. .\
. . 1. rel()
. . 2. Term List
. . 3. u : Term
. . 4. v : Term List
. . 5. ∀i:ℕ.
. . |  reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
. . |  i<||v|| ⇒ term_free_vars(v[i]) = nil
. . 6. i : ℕ
. . 7. term_free_vars(u) = nil
. . 8. reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil
. . 9. i<||v||+1
. . 10. ¬i = 0
. . ⊢  term_free_vars([u / v][i]) = nil
. . | by RWW "select_cons_tl" 0 THEN Auto
. . |
. . 1. rel()
. . 2. Term List
. . 3. u : Term
. . 4. v : Term List
. . 5. ∀i:ℕ.
. .    reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
. .    i<||v|| ⇒ term_free_vars(v[i]) = nil
. . 6. i : ℕ
. . 7. term_free_vars(u) = nil
. . 8. reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil
. . 9. i<||v||+1
. . 10. ¬i = 0
. . ⊢  term_free_vars(v[(i-1)]) = nil  by BackThruSomeHyp THEN Auto
END
```