```
closed_rel_mng_sq  THM

⊢  ∀r:rel(), rho,ds,da1,da2,de,s,e,a1,a2,tr:Top.
|  closed_rel(r) ⇒
|  ([[r]] rho ds da1 de e s a1 tr ∼ [[r]] rho ds da2 de e s a2 tr)
| by UnivCD THENA Auto
|
1. r : rel()
2. rho : Top
3. ds : Top
4. da1 : Top
5. da2 : Top
6. de : Top
7. s : Top
8. e : Top
9. a1 : Top
10. a2 : Top
11. tr : Top
12. closed_rel(r)
⊢  [[r]] rho ds da1 de e s a1 tr ∼ [[r]] rho ds da2 de e s a2 tr
| by MoveToConcl (-1) THEN

|    Unfolds ``closed_rel rel_mng`` 0 THEN Unfold `rel_free_vars` 0

|    THEN GenConcl r.args = l THENA Auto
THEN GenConcl
|    [[r.name]] rho 2of(e)  = X THENA Auto
|•
|
1. r : rel()
2. rho : Top
3. Top
4. Top
5. Top
6. Top
7. s : Top
8. e : Top
9. a1 : Top
10. a2 : Top
11. tr : Top
12. l : Term List
13. r.args = l
14. X : Top
15. [[r.name]] rho 2of(e)  = X
⊢  reduce(λt,vs. term_free_vars(t) @ vs;nil;l) = nil ⇒
|  (list_accum(x,t.x
|                  ([[t]] 1of(e) s a1 tr);X;l) ∼ ...)
| by ...
|
|\
| 1. Top
| 2. Top
| 3. Top
| 4. Top
| 5. Top
```

```
| 6. Term List
| ⊢  ∀X:Top. nil = nil ⇒ (X ∼ X)  by Auto
|
.\
. 1. s : Top
. 2. e : Top
. 3. a1 : Top
. 4. a2 : Top
. 5. tr : Top
. 6. Term List
. 7. u : Term
. 8. v : Term List
. 9. ∀X:Top.
. |  reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
. |  (list_accum(x,t.x
. |                 ([[t]] 1of(e) s a1 tr);X;v) ∼ ...)
. ⊢  ∀X:Top.
. |  (term_free_vars(u) @ reduce(λt,vs. term_free_vars(t) @ vs;nil;v))
. |  =
. |  nil
. |  ⇒
. |
. |  (list_accum(x,t.x
. |                 ([[t]] 1of(e) s a1 tr);...;v) ∼ ...)
. | by RepeatFor 2 (D 0 THENA Auto)
. |
. 1. s : Top
. 2. e : Top
. 3. a1 : Top
. 4. a2 : Top
. 5. tr : Top
. 6. Term List
. 7. u : Term
. 8. v : Term List
. 9. ∀X:Top.
. |  reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
. |  (list_accum(x,t.x
. |                 ([[t]] 1of(e) s a1 tr);X;v) ∼ ...)
. 10. X : Top
. 11. (term_free_vars(u)
. |    @ reduce(λt,vs. term_free_vars(t) @ vs;nil;v))
. |   =
. |   nil
. ⊢  list_accum(x,t.x
. |                ([[t]] 1of(e) s a1 tr);...;v) ∼ ...
. | by InstHyp [X([[u]] 1of(e) s a1 tr)] (-3) THEN Try Trivial
. |
. |\
. | .....antecedent .....
. | 1. s : Top
. | 2. e : Top
. | 3. a1 : Top
. | 4. a2 : Top
. | 5. tr : Top
. | 6. Term List
```

```
. | 7. u : Term
. | 8. v : Term List
. | 9. ∀X:Top.
. |    reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
. |    (list_accum(x,t.x
. |                  ([[t]] 1of(e) s a1 tr);X;v) ∼ ...)
. | 10. Top
. | 11. (term_free_vars(u)
. |      @ reduce(λt,vs. term_free_vars(t) @ vs;nil;v))
. |     =
. |     nil
. | ⊢  reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil
. |   by RWO "append_is_nil" (-1) THEN Auto
. |

. .\
. . 1. s : Top
. . 2. e : Top
. . 3. a1 : Top
. . 4. a2 : Top
. . 5. tr : Top
. . 6. Term List
. . 7. u : Term
. . 8. v : Term List
. . 9. ∀X:Top.
. . |  reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
. . |  (list_accum(x,t.x
. . |                  ([[t]] 1of(e) s a1 tr);X;v) ∼ ...)
. . 10. X : Top
. . 11. (term_free_vars(u)
. . |    @ reduce(λt,vs. term_free_vars(t) @ vs;nil;v))
. . |   =
. . |   nil
. . 12. list_accum(x,t.x
. . |                  ([[t]] 1of(e) s a1 tr);...;v) ∼ ...
. . ⊢  list_accum(x,t.x
. . |                 ([[t]] 1of(e) s a1 tr);...;v) ∼ ...
. . | by Subst [[u]] 1of(e) s a2 tr ∼ [[u]] 1of(e) s a1 tr
. . |     0 THEN Try Trivial
. . |
. . .....equality .....
. . 1. s : Top
. . 2. e : Top
. . 3. a1 : Top
. . 4. a2 : Top
. . 5. tr : Top
. . 6. Term List
. . 7. u : Term
. . 8. v : Term List
. . 9. ∀X:Top.
. . |  reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
. . |  (list_accum(x,t.x
. . |                  ([[t]] 1of(e) s a1 tr);X;v) ∼ ...)
. . 10. X : Top
. . 11. (term_free_vars(u)
. . |    @ reduce(λt,vs. term_free_vars(t) @ vs;nil;v))
```

```
. . |   =
. . |   nil
. . 12. list_accum(x,t.x
. . |                  ([[t]] 1of(e) s a1 tr);...;v) ∼ ...
. . ⊢  [[u]] 1of(e) s a2 tr ∼ [[u]] 1of(e) s a1 tr
. . | by Thin (-1) THEN BackThruLemma ‘closed_term_mng‘
. .
. . |    THEN Auto THEN RWO "append_is_nil" (-1) THEN Auto
. . |
. . 1. s : Top
. . 2. e : Top
. . 3. a1 : Top
. . 4. a2 : Top
. . 5. tr : Top
. . 6. Term List
. . 7. u : Term
. . 8. v : Term List
. . 9. ∀X:Top.
. .    reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil ⇒
. .    (list_accum(x,t.x
. .                  ([[t]] 1of(e) s a1 tr);X;v) ∼ ...)
. . 10. Top
. . 11. term_free_vars(u) = nil
. . 12. reduce(λt,vs. term_free_vars(t) @ vs;nil;v) = nil
. . ⊢  closed_term(u)  by Unfold ‘closed_term‘ 0 THEN Auto
END
```