```
member_col_subst  THM

⊢  ∀c:(Label→Collection(Term)), r,r':rel().
|  r' ∈ col_subst(c;r)
|  ⟺
|  (∃as:(Label×Term) List.
|   1of(unzip(as)) = rel_vars(r)
|   & (∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i])))
|   & r' = rel_subst(as;r))
| by Unfold `col_subst` 0 THEN
Unfold `col_map_subst` 0 THEN

|    RWW "member_col_map" 0
THENM RWW "member_col_list_prod" 0

|    THEN Auto'
THEN ExRepD
THEN InstConcl [as]
THEN Auto'

|    THEN AllHyps (RWW "map_length_nat") THEN Auto

|    THEN Try (SubstFor as 0 THEN RWW "unzip_zip" 0 THENM Reduce 0

|              THEN Complete Auto)
|
|
|\
| 1. c : Label→Collection(Term)
| 2. r : rel()
| 3. r' : rel()
| 4. as : (Label×Term) List
| 5. s : Term List
| 6. ||s|| = ||rel_vars(r)||
| 7. ∀i:ℕ. i<||s|| ⇒ s[i] ∈ map(c;rel_vars(r))[i]
| 8. as = zip(rel_vars(r);s)
| 9. r' = rel_subst(as;r)
| 10. i : ℕ
| 11. i<||as||
| ⊢  2of(as[i]) ∈ c(1of(as[i]))
| | by MoveToConcl (-1) THEN SubstFor as 0
| THENM RWW "select_zip" 0
|
| |    THENM Reduce 0
| THENM D 0
| THENM RWW "length_zip" (-1)
|
| |    THENA Auto
| |
| 1. c : Label→Collection(Term)
| 2. r : rel()
| 3. r' : rel()
| 4. as : (Label×Term) List
| 5. s : Term List
| 6. ||s|| = ||rel_vars(r)||
```

```
| 7. ∀i:ℕ. i<||s|| ⇒ s[i] ∈ map(c;rel_vars(r))[i]
| 8. as = zip(rel_vars(r);s)
| 9. r' = rel_subst(as;r)
| 10. i : ℕ
| 11. i<||rel_vars(r)||
| ⊢  s[i] ∈ c(rel_vars(r)[i])
|   by InstHyp [i] (-5) THENA Auto'
| THEN RWW "map_select" (-1)
|
|      THEN Auto
|
.\
. 1. c : Label→Collection(Term)
. 2. r : rel()
. 3. r' : rel()
. 4. as : (Label×Term) List
. 5. 1of(unzip(as)) = rel_vars(r)
. 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. 7. r' = rel_subst(as;r)
. ⊢  ∃s:Term List.
. |  ||s|| = ||map(c;rel_vars(r))||
. |  & (∀i:ℕ. i<||s|| ⇒ s[i] ∈ map(c;rel_vars(r))[i])
. |  & as = zip(rel_vars(r);s)
. | by InstConcl [2of(unzip(as))] THEN Auto'
. |
. |\
. | 1. c : Label→Collection(Term)
. | 2. r : rel()
. | 3. r' : rel()
. | 4. as : (Label×Term) List
. | 5. 1of(unzip(as)) = rel_vars(r)
. | 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | 7. r' = rel_subst(as;r)
. | ⊢  ||2of(unzip(as))|| = ||map(c;rel_vars(r))||
. |   by All (\i. Unfold `unzip` i THEN Reduce i THEN
. |
. |           RWW "map_length_nat" i)
. | THEN Auto
. | THEN SubstFor
. |      rel_vars(r) 0
. | THENM RWW "map_length_nat" 0
. | THEN Auto'
. |
. |\
. | 1. c : Label→Collection(Term)
. | 2. r : rel()
. | 3. r' : rel()
. | 4. as : (Label×Term) List
. | 5. 1of(unzip(as)) = rel_vars(r)
. | 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | 7. r' = rel_subst(as;r)
. | 8. i : ℕ
. | 9. i<||2of(unzip(as))||
. | ⊢  2of(unzip(as))[i] ∈ map(c;rel_vars(r))[i]
. | | by Unfold `unzip` 0 THEN Reduce 0
```

```
. | THENM RWW "map_length_nat" 0
. | |
. | THENM RWW "map_select" 0
. | THEN Auto
. | |
. | |\
. | | 1. c : Label→Collection(Term)
. | | 2. r : rel()
. | | 3. r' : rel()
. | | 4. as : (Label×Term) List
. | | 5. 1of(unzip(as)) = rel_vars(r)
. | | 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | | 7. r' = rel_subst(as;r)
. | | 8. i : ℕ
. | | 9. i<||2of(unzip(as))||
. | | ⊢  i<||as||
. | |   by ...
. | | Auto
. | |
. | |\
. | | 1. c : Label→Collection(Term)
. | | 2. r : rel()
. | | 3. r' : rel()
. | | 4. as : (Label×Term) List
. | | 5. 1of(unzip(as)) = rel_vars(r)
. | | 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | | 7. r' = rel_subst(as;r)
. | | 8. i : ℕ
. | | 9. i<||2of(unzip(as))||
. | | ⊢  i<||rel_vars(r)||  by SubstFor rel_vars(r) 0 THENA Auto
. | | |
. | | 1. c : Label→Collection(Term)
. | | 2. r : rel()
. | | 3. r' : rel()
. | | 4. as : (Label×Term) List
. | | 5. 1of(unzip(as)) = rel_vars(r)
. | | 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | | 7. r' = rel_subst(as;r)
. | | 8. i : ℕ
. | | 9. i<||2of(unzip(as))||
. | | ⊢  i<||1of(unzip(as))||
. | |   by ...
. | | Auto
. | |
. | .\
. | . 1. c : Label→Collection(Term)
. | . 2. r : rel()
. | . 3. r' : rel()
. | . 4. as : (Label×Term) List
. | . 5. 1of(unzip(as)) = rel_vars(r)
. | . 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | . 7. r' = rel_subst(as;r)
. | . 8. i : ℕ
. | . 9. i<||2of(unzip(as))||
. | . ⊢  (λp.2of(p))(as[i]) ∈ c(rel_vars(r)[i])
```

```
. | . | by Reduce 0 THEN Subst rel_vars(r)[i] ~ 1of(as[i]) 0
. | .
. | . |    THEN Try (BackThruSomeHyp THEN
. | .
. | . |              AllHyps (\i. Unfold ‘unzip‘ i THEN Reduce i
. | .
. | . |    ...
. | . |        )
. | . |
. | . |
. | . .....equality .....
. | . 1. c : Label→Collection(Term)
. | . 2. r : rel()
. | . 3. r’ : rel()
. | . 4. as : (Label×Term) List
. | . 5. 1of(unzip(as)) = rel_vars(r)
. | . 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | . 7. r’ = rel_subst(as;r)
. | . 8. i : ℕ
. | . 9. i<||2of(unzip(as))||
. | . ⊢  rel_vars(r)[i] ~ 1of(as[i])
. | . | by Auto THEN Subst rel_vars(r) ~ 1of(unzip(as)) 0 THEN Auto
. | . |
. | . |\
. | . | 1. c : Label→Collection(Term)
. | . | 2. r : rel()
. | . | 3. r’ : rel()
. | . | 4. as : (Label×Term) List
. | . | 5. 1of(unzip(as)) = rel_vars(r)
. | . | 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | . | 7. r’ = rel_subst(as;r)
. | . | 8. i : ℕ
. | . | 9. i<||2of(unzip(as))||
. | . | 10. rel_vars(r) = 1of(unzip(as))
. | . | 11. x : Label List
. | . | 12. y : Label List
. | . | 13. x = y
. | . | ⊢  x ~ y  by Assert SQType(Label List)
. | . | |
. | . | |\
. | . | | .....assertion .....
. | . | | 1. c : Label→Collection(Term)
. | . | | 2. r : rel()
. | . | | 3. r’ : rel()
. | . | | 4. as : (Label×Term) List
. | . | | 5. 1of(unzip(as)) = rel_vars(r)
. | . | | 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | . | | 7. r’ = rel_subst(as;r)
. | . | | 8. i : ℕ
. | . | | 9. i<||2of(unzip(as))||
. | . | | 10. rel_vars(r) = 1of(unzip(as))
. | . | | 11. x : Label List
. | . | | 12. y : Label List
. | . | | 13. x = y
. | . | | ⊢  SQType(Label List)  by SqTac
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. | . | |
. | . | .\
. | . | . 1. c : Label→Collection(Term)
. | . | . 2. r : rel()

. | . | . 3. r' : rel()
. | . | . 4. as : (Label×Term) List
. | . | . 5. 1of(unzip(as)) = rel_vars(r)
. | . | . 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | . | . 7. r' = rel_subst(as;r)
. | . | . 8. i : ℕ
. | . | . 9. i<||2of(unzip(as))||
. | . | . 10. rel_vars(r) = 1of(unzip(as))
. | . | . 11. x : Label List
. | . | . 12. y : Label List
. | . | . 13. x = y
. | . | . 14. SQType(Label List)
. | . | . ⊢  x ~ y
. | . | .   by Unfold `guard` 0 THEN Unfold `sq_type` (-1)
. | . | .
. | . | .      THEN BackThruSomeHyp THEN Auto
. | . |
. | . .\
. | . . 1. c : Label→Collection(Term)
. | . . 2. r : rel()
. | . . 3. r' : rel()
. | . . 4. as : (Label×Term) List
. | . . 5. 1of(unzip(as)) = rel_vars(r)
. | . . 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. | . . 7. r' = rel_subst(as;r)
. | . . 8. i : ℕ
. | . . 9. i<||2of(unzip(as))||
. | . . ⊢  1of(unzip(as))[i] = 1of(as[i])
. | . .   by MoveToConcl (-1) THEN Unfold `unzip` 0 THEN Reduce 0
. | . .
. | . .      THEN RWW "map_select" 0 THEN Reduce 0 THEN Auto
. | . .
. | . .      THEN RWW "map_length_nat" (-1) THEN Auto
. |
. .\
. . 1. c : Label→Collection(Term)
. . 2. r : rel()
. . 3. r' : rel()
. . 4. as : (Label×Term) List
. . 5. 1of(unzip(as)) = rel_vars(r)
. . 6. ∀i:ℕ. i<||as|| ⇒ 2of(as[i]) ∈ c(1of(as[i]))
. . 7. r' = rel_subst(as;r)
. . ⊢  as = zip(rel_vars(r);2of(unzip(as)))
. .   by SubstFor rel_vars(r) 0
. . THENM RWW "zip_unzip" 0
. . THEN Auto
END
```