```
rel_subst_tc  THM

⊢  ∀r:rel(), as:(Label×Term) List, ds:Collection(dec())
|  , da:Collection(SimpleType), de:sig().
|  tc(r;ds;da;de) ⇒
|  (∀x:Label.
|   (x ∈ rel_vars(r)) ⇒
|   (∀t:SimpleType.
|    mk_dec(x, t) ∈ ds ⇒
|    t ∈ term_types(ds;da;de;apply_alist(as;x;x))))
|  ⇒
|  tc(rel_subst(as;r);ds;da;de)  by UnivCD THENA Auto
|
1. r : rel()
2. as : (Label×Term) List
3. ds : Collection(dec())
4. da : Collection(SimpleType)
5. de : sig()
6. tc(r;ds;da;de)
7. ∀x:Label.
|  (x ∈ rel_vars(r)) ⇒
|  (∀t:SimpleType.
|   mk_dec(x, t) ∈ ds ⇒ t ∈ term_types(ds;da;de;apply_alist(as;x;x)))
⊢  tc(rel_subst(as;r);ds;da;de)
| by All (Unfolds ‘‘tc rel_subst‘‘) THEN D 1 THEN D 1 THEN All Reduce
|
THEN ExRepD
|
|\
| 1. x : SimpleType
| 2. r1 : Term List
| 3. as : (Label×Term) List
| 4. ds : Collection(dec())
| 5. da : Collection(SimpleType)
| 6. de : sig()
| 7. ||r1|| = 2
| 8. x ∈ term_types(ds;da;de;r1[0])
| 9. x ∈ term_types(ds;da;de;r1[1])
| 10. ∀x@0:Label.
| |   (x@0 ∈ rel_vars(<inl(x),r1>)) ⇒
| |   (∀t:SimpleType.
| |    mk_dec(x@0, t) ∈ ds ⇒
| |    t ∈ term_types(ds;da;de;apply_alist(as;x@0;x@0)))
| ⊢  ||map(λt.term_subst(as;t);r1)|| = 2
| |  & x ∈ term_types(ds;da;de;map(λt.term_subst(as;t);r1)[0])
| |    & x ∈ term_types(ds;da;de;map(λt.term_subst(as;t);r1)[1])
| | by D 0
| |
| |\
| | 1. x : SimpleType
| | 2. r1 : Term List
| | 3. as : (Label×Term) List
| | 4. ds : Collection(dec())
| | 5. da : Collection(SimpleType)
| | 6. de : sig()
```

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| | 7. ||r1|| = 2
| | 8. x ∈ term_types(ds;da;de;r1[0])
| | 9. x ∈ term_types(ds;da;de;r1[1])
| | 10. ∀x@0:Label.
| |     (x@0 ∈ rel_vars(<inl(x),r1>)) ⇒
| |     (∀t:SimpleType.
| |      mk_dec(x@0, t) ∈ ds ⇒
| |      t ∈ term_types(ds;da;de;apply_alist(as;x@0;x@0)))
| | ⊢  ||map(λt.term_subst(as;t);r1)|| = 2
| |   by RWW "map_length" 0 THEN Auto
| |
| .\
| . 1. x : SimpleType
| . 2. r1 : Term List
| . 3. as : (Label×Term) List
| . 4. ds : Collection(dec())
| . 5. da : Collection(SimpleType)
| . 6. de : sig()
| . 7. ||r1|| = 2
| . 8. x ∈ term_types(ds;da;de;r1[0])
| . 9. x ∈ term_types(ds;da;de;r1[1])
| . 10. ∀x@0:Label.
| . |   (x@0 ∈ rel_vars(<inl(x),r1>)) ⇒
| . |   (∀t:SimpleType.
| . |    mk_dec(x@0, t) ∈ ds ⇒
| . |    t ∈ term_types(ds;da;de;apply_alist(as;x@0;x@0)))
| . 11. ||map(λt.term_subst(as;t);r1)|| = 2
| . ⊢  x ∈ term_types(ds;da;de;map(λt.term_subst(as;t);r1)[0])
| . |  & x ∈ term_types(ds;da;de;map(λt.term_subst(as;t);r1)[1])
| . | by RWW "map_length" (-1)
| . THENM RWW "map_select" 0
| .
| . |    THENM Reduce 0
| . |THENA Auto
| . |
| . 1. x : SimpleType
| . 2. r1 : Term List
| . 3. as : (Label×Term) List
| . 4. ds : Collection(dec())
| . 5. da : Collection(SimpleType)
| . 6. de : sig()
| . 7. ||r1|| = 2
| . 8. x ∈ term_types(ds;da;de;r1[0])
| . 9. x ∈ term_types(ds;da;de;r1[1])
| . 10. ∀x@0:Label.
| . |   (x@0 ∈ rel_vars(<inl(x),r1>)) ⇒
| . |   (∀t:SimpleType.
| . |    mk_dec(x@0, t) ∈ ds ⇒
| . |    t ∈ term_types(ds;da;de;apply_alist(as;x@0;x@0)))
| . 11. ||r1|| = 2
| . ⊢  x ∈ term_types(ds;da;de;term_subst(as;r1[0]))
| . |  & x ∈ term_types(ds;da;de;term_subst(as;r1[1]))
| . | by D 0 THEN BackThruLemma ‘term_subst_tc‘ THEN Auto
| .
| . |    THEN InstHyp [x1] (-6) THENA Auto
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| .
| . |    THEN Try (BackThruSomeHyp THEN Auto)
| .
| . |    THEN Try (Folds ``relname_eq mk_rel`` 0 THEN
| .
| . |              RWW "member_rel_vars" 0 THEN Auto THEN
| .
| . |              Reduce 0 )
| . |
| . |
| . |\
| . | 1. x : SimpleType
| . | 2. r1 : Term List
| . | 3. as : (Label×Term) List
| . | 4. ds : Collection(dec())
| . | 5. da : Collection(SimpleType)
| . | 6. de : sig()
| . | 7. ||r1|| = 2
| . | 8. x ∈ term_types(ds;da;de;r1[0])
| . | 9. x ∈ term_types(ds;da;de;r1[1])
| . | 10. ∀x@0:Label.
| . |     (x@0 ∈ rel_vars(<inl(x),r1>)) ⇒
| . |     (∀t:SimpleType.
| . |      mk_dec(x@0, t) ∈ ds ⇒
| . |      t ∈ term_types(ds;da;de;apply_alist(as;x@0;x@0)))
| . | 11. ||r1|| = 2
| . | 12. x1 : Label
| . | 13. (x1 ∈ term_vars(r1[0]))
| . | 14. t : SimpleType
| . | 15. mk_dec(x1, t) ∈ ds
| . | ⊢  ∃i:ℕ. i<||r1|| & (x1 ∈ term_vars(r1[i]))
| . |   by InstConcl [0] THEN Auto
| . |
| . .\
| . . 1. x : SimpleType
| . . 2. r1 : Term List
| . . 3. as : (Label×Term) List
| . . 4. ds : Collection(dec())
| . . 5. da : Collection(SimpleType)
| . . 6. de : sig()
| . . 7. ||r1|| = 2
| . . 8. x ∈ term_types(ds;da;de;r1[0])
| . . 9. x ∈ term_types(ds;da;de;r1[1])
| . . 10. ∀x@0:Label.
| . .     (x@0 ∈ rel_vars(<inl(x),r1>)) ⇒
| . .     (∀t:SimpleType.
| . .      mk_dec(x@0, t) ∈ ds ⇒
| . .      t ∈ term_types(ds;da;de;apply_alist(as;x@0;x@0)))
| . . 11. ||r1|| = 2
| . . 12. x1 : Label
| . . 13. (x1 ∈ term_vars(r1[1]))
| . . 14. t : SimpleType
| . . 15. mk_dec(x1, t) ∈ ds
| . . ⊢  ∃i:ℕ. i<||r1|| & (x1 ∈ term_vars(r1[i]))
| . .   by InstConcl [1] THEN Auto
```

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|
.\
. 1. y : Label
. 2. r1 : Term List
. 3. as : (Label×Term) List
. 4. ds : Collection(dec())
. 5. da : Collection(SimpleType)
. 6. de : sig()
. 7. ||de.rel(y)|| = ||r1||
. 8. ∀i:ℕ. i<||r1|| ⇒ (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. 9. ∀x:Label.
. |  (x ∈ rel_vars(<inr(y),r1>)) ⇒
. |  (∀t:SimpleType.
. |   mk_dec(x, t) ∈ ds ⇒
. |   t ∈ term_types(ds;da;de;apply_alist(as;x;x)))
. ⊢  ||de.rel(y)|| = ||map(λt.term_subst(as;t);r1)||
. |  & (∀i:ℕ.
. |     i<||map(λt.term_subst(as;t);r1)|| ⇒
. |     (de.rel(y))[i] ∈ term_types(ds;da;de;map(λt.
. |     term_subst(as;t);r1)[i]))  by D 0
. |
. |\
. | 1. y : Label
. | 2. r1 : Term List
. | 3. as : (Label×Term) List
. | 4. ds : Collection(dec())
. | 5. da : Collection(SimpleType)
. | 6. de : sig()
. | 7. ||de.rel(y)|| = ||r1||
. | 8. ∀i:ℕ. i<||r1|| ⇒ (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. | 9. ∀x:Label.
. |    (x ∈ rel_vars(<inr(y),r1>)) ⇒
. |    (∀t:SimpleType.
. |     mk_dec(x, t) ∈ ds ⇒
. |     t ∈ term_types(ds;da;de;apply_alist(as;x;x)))
. | ⊢  ||de.rel(y)|| = ||map(λt.term_subst(as;t);r1)||
. |   by RWW "map_length_nat" 0 THEN Auto

. |
. .\
. . 1. y : Label
. . 2. r1 : Term List
. . 3. as : (Label×Term) List
. . 4. ds : Collection(dec())
. . 5. da : Collection(SimpleType)
. . 6. de : sig()
. . 7. ||de.rel(y)|| = ||r1||
. . 8. ∀i:ℕ. i<||r1|| ⇒ (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . 9. ∀x:Label.
. . |  (x ∈ rel_vars(<inr(y),r1>)) ⇒
. . |  (∀t:SimpleType.
. . |   mk_dec(x, t) ∈ ds ⇒
. . |   t ∈ term_types(ds;da;de;apply_alist(as;x;x)))
. . 10. ||de.rel(y)|| = ||map(λt.term_subst(as;t);r1)||
. . ⊢  ∀i:ℕ.
. . |  i<||map(λt.term_subst(as;t);r1)|| ⇒
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. . |  (de.rel(y))[i] ∈ term_types(ds;da;de;map(λt.
. . |  term_subst(as;t);r1)[i])
. . | by RWW "map_length_nat" (-1)
. . THENM UnivCD
. .
. . |    THENM RWW "map_length" (-1)
. . THENM RWW "map_select" 0
. .
. . |    THENM Reduce 0
. . THENA Auto
. . |
. . 1. y : Label
. . 2. r1 : Term List
. . 3. as : (Label×Term) List
. . 4. ds : Collection(dec())
. . 5. da : Collection(SimpleType)
. . 6. de : sig()
. . 7. ||de.rel(y)|| = ||r1||
. . 8. ∀i:ℕ. i<||r1|| ⇒ (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . 9. ∀x:Label.
. . |  (x ∈ rel_vars(<inr(y),r1>)) ⇒
. . |  (∀t:SimpleType.
. . |   mk_dec(x, t) ∈ ds ⇒
. . |   t ∈ term_types(ds;da;de;apply_alist(as;x;x)))
. . 10. ||de.rel(y)|| = ||r1||
. . 11. i : ℕ
. . 12. i<||r1||
. . ⊢  (de.rel(y))[i] ∈ term_types(ds;da;de;term_subst(as;r1[i]))
. . | by InstHyp [i] (-5) THENA Auto
. . |
. . 1. y : Label
. . 2. r1 : Term List
. . 3. as : (Label×Term) List
. . 4. ds : Collection(dec())
. . 5. da : Collection(SimpleType)
. . 6. de : sig()
. . 7. ||de.rel(y)|| = ||r1||
. . 8. ∀i:ℕ. i<||r1|| ⇒ (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . 9. ∀x:Label.
. . |  (x ∈ rel_vars(<inr(y),r1>)) ⇒
. . |  (∀t:SimpleType.
. . |   mk_dec(x, t) ∈ ds ⇒
. . |   t ∈ term_types(ds;da;de;apply_alist(as;x;x)))
. . 10. ||de.rel(y)|| = ||r1||
. . 11. i : ℕ
. . 12. i<||r1||
. . 13. (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . ⊢  (de.rel(y))[i] ∈ term_types(ds;da;de;term_subst(as;r1[i]))
. . | by BackThruLemma ‘term_subst_tc‘ THEN Auto
. . |
. . 1. y : Label
. . 2. r1 : Term List
. . 3. as : (Label×Term) List
. . 4. ds : Collection(dec())
. . 5. da : Collection(SimpleType)
```

```
. . 6. de : sig()
. . 7. ||de.rel(y)|| = ||r1||
. . 8. ∀i:ℕ. i<||r1|| ⇒ (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . 9. ∀x:Label.
. . |  (x ∈ rel_vars(<inr(y),r1>)) ⇒
. . |  (∀t:SimpleType.
. . |   mk_dec(x, t) ∈ ds ⇒
. . |   t ∈ term_types(ds;da;de;apply_alist(as;x;x)))
. . 10. ||de.rel(y)|| = ||r1||
. . 11. i : ℕ
. . 12. i<||r1||
. . 13. (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . 14. x : Label
. . 15. (x ∈ term_vars(r1[i]))
. . 16. t : SimpleType
. . 17. mk_dec(x, t) ∈ ds
. . ⊢  t ∈ term_types(ds;da;de;apply_alist(as;x;x))
. . | by InstHyp [x] (-9) THEN Auto
. . |
. . |\
. . | .....antecedent .....
. . | 1. y : Label
. . | 2. r1 : Term List
. . | 3. as : (Label×Term) List
. . | 4. ds : Collection(dec())
. . | 5. da : Collection(SimpleType)
. . | 6. de : sig()
. . | 7. ||de.rel(y)|| = ||r1||
. . | 8. ∀i:ℕ. i<||r1|| ⇒ (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . | 9. ∀x:Label.
. . | |  (x ∈ rel_vars(<inr(y),r1>)) ⇒
. . | |  (∀t:SimpleType.
. . | |   mk_dec(x, t) ∈ ds ⇒
. . | |   t ∈ term_types(ds;da;de;apply_alist(as;x;x)))
. . | 10. ||de.rel(y)|| = ||r1||
. . | 11. i : ℕ
. . | 12. i<||r1||
. . | 13. (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . | 14. x : Label
. . | 15. (x ∈ term_vars(r1[i]))
. . | 16. t : SimpleType
. . | 17. mk_dec(x, t) ∈ ds
. . | ⊢  (x ∈ rel_vars(<inr(y),r1>))
. . | | by Folds ‘‘relname_other mk_rel‘‘ 0
. . |
. . | |    THEN RWW "member_rel_vars" 0
. . | THEN Auto THEN Reduce 0
. . | |
. . | 1. y : Label
. . | 2. r1 : Term List
. . | 3. as : (Label×Term) List
. . | 4. ds : Collection(dec())
. . | 5. da : Collection(SimpleType)
. . | 6. de : sig()
. . | 7. ||de.rel(y)|| = ||r1||
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. . | 8. ∀i:ℕ. i<||r1|| ⇒ (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . | 9. ∀x:Label.
. . |    (x ∈ rel_vars(<inr(y),r1>)) ⇒
. . |    (∀t:SimpleType.
. . |     mk_dec(x, t) ∈ ds ⇒
. . |     t ∈ term_types(ds;da;de;apply_alist(as;x;x)))
. . | 10. ||de.rel(y)|| = ||r1||
. . | 11. i : ℕ
. . | 12. i<||r1||
. . | 13. (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . | 14. x : Label
. . | 15. (x ∈ term_vars(r1[i]))
. . | 16. t : SimpleType
. . | 17. mk_dec(x, t) ∈ ds
. . | ⊢  ∃i:ℕ. i<||r1|| & (x ∈ term_vars(r1[i]))
. . |   by InstConcl [i] THEN Auto
. . |
. . .\
. . . 1. y : Label
. . . 2. r1 : Term List
. . . 3. as : (Label×Term) List
. . . 4. ds : Collection(dec())
. . . 5. da : Collection(SimpleType)
. . . 6. de : sig()
. . . 7. ||de.rel(y)|| = ||r1||
. . . 8. ∀i:ℕ. i<||r1|| ⇒ (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . . 9. ∀x:Label.
. . .    (x ∈ rel_vars(<inr(y),r1>)) ⇒
. . .    (∀t:SimpleType.
. . .     mk_dec(x, t) ∈ ds ⇒
. . .     t ∈ term_types(ds;da;de;apply_alist(as;x;x)))
. . . 10. ||de.rel(y)|| = ||r1||
. . . 11. i : ℕ
. . . 12. i<||r1||
. . . 13. (de.rel(y))[i] ∈ term_types(ds;da;de;r1[i])
. . . 14. x : Label
. . . 15. (x ∈ term_vars(r1[i]))
. . . 16. t : SimpleType
. . . 17. mk_dec(x, t) ∈ ds
. . . 18. ∀t:SimpleType.
. . .     mk_dec(x, t) ∈ ds ⇒
. . .     t ∈ term_types(ds;da;de;apply_alist(as;x;x))
. . . ⊢  t ∈ term_types(ds;da;de;apply_alist(as;x;x))
. . .   by BackThruSomeHyp THEN Auto
END
```