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term_subst2_mentions_guard  THM

⊢  ∀as:(Label×Term) List, g:Label, t:Term.
|  ¬subst_mentions_trace(as) ⇒
|  term_mentions_guard(g;term_subst2(as;t)) ⇒
|  term_mentions_guard(g;t)
| by RepeatFor 3 (D 0 THENA Auto) THEN TermInd (-1) THEN Reduce 0

|    THEN Auto

|    ...
|         THEN Complete EasyHyp)

|    ...
|
1. as : (Label×Term) List
2. g : Label
3. Term
4. u : Term→Type
5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
|  term_mentions_guard(g;term_subst2(as;t)) ⇒
|  term_mentions_guard(g;t)
6. x : Label
7. term_mentions_guard(g;apply_alist(as;x;x'))
⊢  ∃i:ℕ||as||. mentions_trace(2of(as[i]))
| by InstLemma `apply_alist_property` [Term;as;x';x] THENA Auto

|    THEN D (-1)
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| 1. as : (Label×Term) List
| 2. g : Label
| 3. Term
| 4. u : Term→Type
| 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| |  term_mentions_guard(g;term_subst2(as;t)) ⇒
| |  term_mentions_guard(g;t)
| 6. x : Label
| 7. term_mentions_guard(g;apply_alist(as;x;x'))
| 8. (apply_alist(as;x;x') ∈ 2of(unzip(as)))
| ⊢  ∃i:ℕ||as||. mentions_trace(2of(as[i]))
| | by Unfold `l_member` (-1) THEN ParallelOp (-1) THEN ExRepD
| |
| |\
| | 1. as : (Label×Term) List
| | 2. g : Label
| | 3. Term
| | 4. u : Term→Type
| | 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| |    term_mentions_guard(g;term_subst2(as;t)) ⇒
| |    term_mentions_guard(g;t)
| | 6. x : Label
| | 7. term_mentions_guard(g;apply_alist(as;x;x'))
| | 8. i : ℕ
| | 9. i<||2of(unzip(as))||
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| | 10. apply_alist(as;x;x') = 2of(unzip(as))[i]
| | ⊢  i<||as||
| |   by ...
| |
| .\
| . 1. as : (Label×Term) List
| . 2. g : Label
| . 3. Term
| . 4. u : Term→Type
| . 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| . |  term_mentions_guard(g;term_subst2(as;t)) ⇒
| . |  term_mentions_guard(g;t)
| . 6. x : Label
| . 7. term_mentions_guard(g;apply_alist(as;x;x'))
| . 8. i : ℕ
| . 9. i<||2of(unzip(as))||
| . 10. apply_alist(as;x;x') = 2of(unzip(as))[i]
| . ⊢  mentions_trace(2of(as[i]))
| . | by Unfold `unzip` (-2) THEN Reduce (-2) THEN
| .
| . |    RWO "map_length" (-2) THENA Auto THEN
| .
| . |    MoveToConcl (-4) THEN HypSubstSq (-1) 0
| . THEN Subst'
| . |    2of(unzip(as))[i] = 2of(as[i]) 0 THENA Auto
| . |
| . |\
| . | 1. as : (Label×Term) List
| . | 2. g : Label
| . | 3. Term
| . | 4. u : Term→Type
| . | 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| . |    term_mentions_guard(g;term_subst2(as;t)) ⇒
| . |    term_mentions_guard(g;t)
| . | 6. x : Label
| . | 7. i : ℕ
| . | 8. i<||as||
| . | 9. apply_alist(as;x;x') = 2of(unzip(as))[i]
| . | 10. ∀x,y:Term. x = y ⇒ (x ∼ y)
| . | ⊢  2of(unzip(as))[i] = 2of(as[i])
| . |   by ...
| . | THEN Reduce 0 THEN Auto
| . |
| . .\
| . . 1. as : (Label×Term) List
| . . 2. g : Label
| . . 3. Term
| . . 4. u : Term→Type
| . . 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| . .    term_mentions_guard(g;term_subst2(as;t)) ⇒
| . .    term_mentions_guard(g;t)
| . . 6. x : Label
| . . 7. i : ℕ
| . . 8. i<||as||
| . . 9. apply_alist(as;x;x') = 2of(unzip(as))[i]
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| . . ⊢  term_mentions_guard(g;2of(as[i])) ⇒
| . .    mentions_trace(2of(as[i]))
| . .   by ...
|
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. 1. as : (Label×Term) List
. 2. g : Label
. 3. Term
. 4. u : Term→Type
. 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
.    term_mentions_guard(g;term_subst2(as;t)) ⇒
.    term_mentions_guard(g;t)
. 6. x : Label
. 7. term_mentions_guard(g;apply_alist(as;x;x'))
. 8. apply_alist(as;x;x') = x'
. ⊢  ∃i:ℕ||as||. mentions_trace(2of(as[i]))
.   by HypSubstSq (-1) (-2) THEN Reduce (-2) THEN Auto

END
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