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term_subst_mentions_guard  THM

⊢  ∀as:(Label×Term) List, g:Label, t:Term.
|  ¬subst_mentions_trace(as) ⇒
|  term_mentions_guard(g;term_subst(as;t)) ⇒ term_mentions_guard(g;t)
| by RepeatFor 3 (D 0 THENA Auto) THEN TermInd (-1) THEN Reduce 0

|    THEN Auto

|    ...
|         THEN Complete EasyHyp)

|    ...
|
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| 1. as : (Label×Term) List
| 2. g : Label
| 3. Term
| 4. u : Term→Type
| 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| |  term_mentions_guard(g;term_subst(as;t)) ⇒
| |  term_mentions_guard(g;t)
| 6. x1 : Label
| 7. term_mentions_guard(g;apply_alist(as;x1;x1))
| ⊢  ∃i:ℕ||as||. mentions_trace(2of(as[i]))
| | by InstLemma `apply_alist_property` [Term;as;x1;x1] THENA Auto
|
| |    THEN D (-1)
| |
| |\
| | 1. as : (Label×Term) List
| | 2. g : Label
| | 3. Term
| | 4. u : Term→Type
| | 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| | |  term_mentions_guard(g;term_subst(as;t)) ⇒
| | |  term_mentions_guard(g;t)
| | 6. x1 : Label
| | 7. term_mentions_guard(g;apply_alist(as;x1;x1))
| | 8. (apply_alist(as;x1;x1) ∈ 2of(unzip(as)))
| | ⊢  ∃i:ℕ||as||. mentions_trace(2of(as[i]))
| | | by Unfold `l_member` (-1) THEN ParallelOp (-1) THEN ExRepD
| | |
| | |\
| | | 1. as : (Label×Term) List
| | | 2. g : Label
| | | 3. Term
| | | 4. u : Term→Type
| | | 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| | |    term_mentions_guard(g;term_subst(as;t)) ⇒
| | |    term_mentions_guard(g;t)
| | | 6. x1 : Label
| | | 7. term_mentions_guard(g;apply_alist(as;x1;x1))
| | | 8. i : ℕ
| | | 9. i<||2of(unzip(as))||
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| | | 10. apply_alist(as;x1;x1) = 2of(unzip(as))[i]
| | | ⊢  i<||as||
| | |   by ...
| | |
| | .\
| | . 1. as : (Label×Term) List
| | . 2. g : Label
| | . 3. Term
| | . 4. u : Term→Type
| | . 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| | . |  term_mentions_guard(g;term_subst(as;t)) ⇒
| | . |  term_mentions_guard(g;t)
| | . 6. x1 : Label
| | . 7. term_mentions_guard(g;apply_alist(as;x1;x1))
| | . 8. i : ℕ
| | . 9. i<||2of(unzip(as))||
| | . 10. apply_alist(as;x1;x1) = 2of(unzip(as))[i]
| | . ⊢  mentions_trace(2of(as[i]))
| | . | by Unfold `unzip` (-2) THEN Reduce (-2) THEN
| | .
| | . |    RWO "map_length" (-2) THENA Auto THEN
| | .
| | . |    MoveToConcl (-4) THEN HypSubstSq (-1) 0
| | . THEN Subst'
| | . |    2of(unzip(as))[i] = 2of(as[i]) 0 THENA Auto
| | . |
| | . |\
| | . | 1. as : (Label×Term) List
| | . | 2. g : Label
| | . | 3. Term
| | . | 4. u : Term→Type
| | . | 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| | . |    term_mentions_guard(g;term_subst(as;t)) ⇒
| | . |    term_mentions_guard(g;t)
| | . | 6. x1 : Label
| | . | 7. i : ℕ
| | . | 8. i<||as||
| | . | 9. apply_alist(as;x1;x1) = 2of(unzip(as))[i]
| | . | 10. ∀x,y:Term. x = y ⇒ (x ∼ y)
| | . | ⊢  2of(unzip(as))[i] = 2of(as[i])
| | . |   by ...
| | . | THEN Reduce 0 THEN Auto
| | . |
| | . .\
| | . . 1. as : (Label×Term) List
| | . . 2. g : Label
| | . . 3. Term
| | . . 4. u : Term→Type
| | . . 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| | . .    term_mentions_guard(g;term_subst(as;t)) ⇒
| | . .    term_mentions_guard(g;t)
| | . . 6. x1 : Label
| | . . 7. i : ℕ
| | . . 8. i<||as||
| | . . 9. apply_alist(as;x1;x1) = 2of(unzip(as))[i]
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| | . . ⊢  term_mentions_guard(g;2of(as[i])) ⇒
| | . .    mentions_trace(2of(as[i]))
| | . .   by ...
| |
| .\
| . 1. as : (Label×Term) List
| . 2. g : Label
| . 3. Term
| . 4. u : Term→Type
| . 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
| .    term_mentions_guard(g;term_subst(as;t)) ⇒
| .    term_mentions_guard(g;t)
| . 6. x1 : Label
| . 7. term_mentions_guard(g;apply_alist(as;x1;x1))
| . 8. apply_alist(as;x1;x1) = x1
| . ⊢  ∃i:ℕ||as||. mentions_trace(2of(as[i]))
| .   by HypSubstSq (-1) (-2) THEN Reduce (-2) THEN Auto
|
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. 1. as : (Label×Term) List
. 2. g : Label
. 3. Term
. 4. u : Term→Type
. 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
. |  term_mentions_guard(g;term_subst(as;t)) ⇒
. |  term_mentions_guard(g;t)
. 6. x : Label
. 7. term_mentions_guard(g;apply_alist(as;x;x'))
. ⊢  ∃i:ℕ||as||. mentions_trace(2of(as[i]))
. | by InstLemma ‘apply_alist_property‘ [Term;as;x';x] THENA Auto
.
. |    THEN D (-1)
. |
. |\
. | 1. as : (Label×Term) List
. | 2. g : Label
. | 3. Term
. | 4. u : Term→Type
. | 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
. | |  term_mentions_guard(g;term_subst(as;t)) ⇒
. | |  term_mentions_guard(g;t)
. | 6. x : Label
. | 7. term_mentions_guard(g;apply_alist(as;x;x'))
. | 8. (apply_alist(as;x;x') ∈ 2of(unzip(as)))
. | ⊢  ∃i:ℕ||as||. mentions_trace(2of(as[i]))
. | | by Unfold ‘l_member‘ (-1) THEN ParallelOp (-1) THEN ExRepD
. | |
. | |\
. | | 1. as : (Label×Term) List
. | | 2. g : Label
. | | 3. Term
. | | 4. u : Term→Type
. | | 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
. | |    term_mentions_guard(g;term_subst(as;t)) ⇒
. | |    term_mentions_guard(g;t)
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. | | 6. x : Label
. | | 7. term_mentions_guard(g;apply_alist(as;x;x'))
. | | 8. i : ℕ
. | | 9. i<||2of(unzip(as))||
. | | 10. apply_alist(as;x;x') = 2of(unzip(as))[i]
. | | ⊢  i<||as||
. | |   by ...
. | |
. | .\
. | . 1. as : (Label×Term) List
. | . 2. g : Label
. | . 3. Term
. | . 4. u : Term→Type
. | . 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
. | . |  term_mentions_guard(g;term_subst(as;t)) ⇒
. | . |  term_mentions_guard(g;t)
. | . 6. x : Label
. | . 7. term_mentions_guard(g;apply_alist(as;x;x'))
. | . 8. i : ℕ
. | . 9. i<||2of(unzip(as))||
. | . 10. apply_alist(as;x;x') = 2of(unzip(as))[i]
. | . ⊢  mentions_trace(2of(as[i]))
. | . | by Unfold `unzip` (-2) THEN Reduce (-2) THEN
. | .
. | . |    RWO "map_length" (-2) THENA Auto THEN
. | .
. | . |    MoveToConcl (-4) THEN HypSubstSq (-1) 0
. | . THEN Subst'
. | . |    2of(unzip(as))[i] = 2of(as[i]) 0 THENA Auto
. | . |
. | . |\
. | . | 1. as : (Label×Term) List
. | . | 2. g : Label
. | . | 3. Term
. | . | 4. u : Term→Type
. | . | 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
. | . |    term_mentions_guard(g;term_subst(as;t)) ⇒
. | . |    term_mentions_guard(g;t)
. | . | 6. x : Label
. | . | 7. i : ℕ
. | . | 8. i<||as||
. | . | 9. apply_alist(as;x;x') = 2of(unzip(as))[i]
. | . | 10. ∀x,y:Term. x = y ⇒ (x ~ y)
. | . | ⊢  2of(unzip(as))[i] = 2of(as[i])
. | . |   by ...
. | . | THEN Reduce 0 THEN Auto
. | . |

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. | . . 1. as : (Label×Term) List
. | . . 2. g : Label
. | . . 3. Term
. | . . 4. u : Term→Type
. | . . 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
. | . . |  term_mentions_guard(g;term_subst(as;t)) ⇒
. | . . |  term_mentions_guard(g;t)
```

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. | . . 6. x : Label
. | . . 7. i : ℕ
. | . . 8. i<||as||
. | . . 9. apply_alist(as;x;x') = 2of(unzip(as))[i]
. | . . ⊢  term_mentions_guard(g;2of(as[i])) ⇒
. | . . |  mentions_trace(2of(as[i]))
. | . . | by GenConcl 2of(as[i]) = t2 THENA Auto THEN Thin (-1) THEN
. | . .
. | . . |    TermInd (-1) THEN Reduce 0 THEN Auto
. | . . |
. | . . 1. as : (Label×Term) List
. | . . 2. g : Label
. | . . 3. Term
. | . . 4. u : Term→Type
. | . . 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
. | . .    term_mentions_guard(g;term_subst(as;t)) ⇒
. | . .    term_mentions_guard(g;t)
. | . . 6. x : Label
. | . . 7. i : ℕ
. | . . 8. i<||as||
. | . . 9. apply_alist(as;x;x') = 2of(unzip(as))[i]
. | . . 10. Term
. | . . 11. u1 : Term→Type
. | . . 12. t2:{v:Term| u1(v) }→term_mentions_guard(g;t2) ⇒
. | . .     mentions_trace(t2)
. | . . 13. y1 : {v:Term| u1(v) }
. | . . 14. y2 : {v:Term| u1(v) }
. | . . 15. term_mentions_guard(g;y1) ∨2 term_mentions_guard(g;y2)
. | . . ⊢  mentions_trace(y1) ∨2 mentions_trace(y2)
. | . .   by ...
. |
. .\
. . 1. as : (Label×Term) List
. . 2. g : Label
. . 3. Term
. . 4. u : Term→Type
. . 5. t:{v:Term| u(v) }→¬subst_mentions_trace(as) ⇒
. .    term_mentions_guard(g;term_subst(as;t)) ⇒
. .    term_mentions_guard(g;t)
. . 6. x : Label
. . 7. term_mentions_guard(g;apply_alist(as;x;x'))
. . 8. apply_alist(as;x;x') = x'
. . ⊢  ∃i:ℕ||as||. mentions_trace(2of(as[i]))
. .   by HypSubstSq (-1) (-2) THEN Reduce (-2) THEN Auto
END
```