```
ioa_all_mng_action  THM

⊢  ∀I:Type, A:(I→ioa{i:l}()), rho:Decl, de:sig(), e:{[[de]] rho}
|  , s:(Σ[[ioa_all(I; i.A(i)).da]] rho), i:I.
|  s ∈ [[A(i)]] rho de e.action  by Auto
|
|\
| 1. I : Type
| 2. A : I→ioa{i:l}()
| 3. rho : Decl
| 4. de : sig()
| 5. e : {[[de]] rho}
| 6. s : (Σ[[ioa_all(I; i.A(i)).da]] rho)
| 7. i : I
| ⊢  s ∈ [[A(i)]] rho de e.action
| | by Unfolds ``ioa_mng`` 0 THEN Reduce 0
| |
| 1. I : Type
| 2. A : I→ioa{i:l}()
| 3. rho : Decl
| 4. de : sig()
| 5. {[[de]] rho}
| 6. s : (Σ[[ioa_all(I; i.A(i)).da]] rho)
| 7. i : I
| ⊢  s ∈ (Σ[[A(i).da]] rho)  by All (Unfold `sigma`) THEN Auto
| |
| .....unproved Inclusion subgoal .....
| 1. I : Type
| 2. A : I→ioa{i:l}()
| 3. rho : Decl
| 4. de : sig()
| 5. {[[de]] rho}
| 6. l:Label×decl_type([[ioa_all(I; i.A(i)).da]] rho;l)
| 7. i : I
| 8. l : Label
| ⊢  decl_type([[ioa_all(I; i.A(i)).da]] rho;l)
| |  ⊆ decl_type([[A(i).da]] rho;l)
| | by Unfolds ``subtype decl_type`` 0 THEN Auto
| |
| |\
| | 1. I : Type
| | 2. A : I→ioa{i:l}()
| | 3. rho : Decl
| | 4. de : sig()
| | 5. {[[de]] rho}
| | 6. l:Label×decl_type([[ioa_all(I; i.A(i)).da]] rho;l)
| | 7. i : I
| | 8. l : Label
| | 9. x : [[ioa_all(I; i.A(i)).da]] rho(l)
| | ⊢  x ∈ [[A(i).da]] rho(l)
| | | by Using [`ds1',ioa_all(I; i.A(i)).da
| | |    ] (BackThruLemma `decls_mng_member`)
| | THEN Auto
| | |
| | 1. I : Type
```

```
| | 2. A : I→ioa{i:l}()
| | 3. rho : Decl
| | 4. de : sig()
| | 5. {[[de]] rho}
| | 6. l:Label×decl_type([[ioa_all(I; i.A(i)).da]] rho;l)
| | 7. i : I
| | 8. l : Label
| | 9. [[ioa_all(I; i.A(i)).da]] rho(l)
| | 10. d : dec()
| | 11. d ∈ A(i).da
| | 12. d.lbl = l
| | ⊢  d ∈ ioa_all(I; i.A(i)).da  by Reduce 0 THEN Auto
| | |
| | 1. I : Type
| | 2. A : I→ioa{i:l}()
| | 3. rho : Decl
| | 4. de : sig()
| | 5. {[[de]] rho}
| | 6. l:Label×decl_type([[ioa_all(I; i.A(i)).da]] rho;l)
| | 7. i : I
| | 8. l : Label
| | 9. [[ioa_all(I; i.A(i)).da]] rho(l)
| | 10. d : dec()
| | 11. d ∈ A(i).da
| | 12. d.lbl = l
| | ⊢  d ∈ ∪i:I. A(i).da
| |   by RWW "member_col_union" 0
| | THENM AutoInstConcl []
| | THENA Auto
| |
| .\
| . .....subterm .....
| . 1. I : Type
| . 2. A : I→ioa{i:l}()
| . 3. rho : Decl
| . 4. de : sig()
| . 5. {[[de]] rho}
| . 6. l:Label×decl_type([[ioa_all(I; i.A(i)).da]] rho;l)
| . 7. I
| . 8. Label
| . ⊢  [[ioa_all(I; i.A(i)).da]] rho ∈ Label→Type
| .   by Fold ‘decl‘ 0 THEN Auto
|
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. .....unproved Inclusion subgoal .....
. 1. I : Type
. 2. I→ioa{i:l}()

. 3. rho : Decl
. 4. de : sig()
. 5. [[de]] rho = [[de]] rho
. 6. Decl×Decl
. ⊢  Decl ⊆ Decl  by Unfold ‘decl‘ 0 THEN Auto
END
```