```
trace_consistent_rel_subst2  THM

⊢  ∀r:rel(), as:(Label×Term) List, daa:Collection(dec()), rho:Decl
|  , te:(Label→Label→𝔹).
|  trace_consistent_rel(rho;daa;te;r)
|  ⇒
|  ¬subst_mentions_trace(as)
|  ⇒
|  trace_consistent_rel(rho;daa;te;rel_subst2(as;r))
| by Unfold ‘rel_subst2‘ 0 THEN Auto

|    THEN RepeatFor 2 (ParallelOp (-2) THEN All Reduce)

|    THEN AssertBY ||map(λt.term_subst2(as;t);r.args)|| = ||r.args||

|       (RWO "map_length" 0 THEN Auto)
|
|\
| 1. r : rel()
| 2. as : (Label×Term) List
| 3. daa : Collection(dec())
| 4. rho : Decl
| 5. te : Label→Label→𝔹
| 6. ∀i:ℕ||r.args||. trace_consistent(rho;daa;te;r.args[i])
| 7. ¬subst_mentions_trace(as)
| 8. i : ℕ||map(λt.term_subst2(as;t);r.args)||
| 9. ||map(λt.term_subst2(as;t);r.args)|| = ||r.args||
| ⊢  i<||r.args||  by Auto
|
.\
. 1. r : rel()
. 2. as : (Label×Term) List
. 3. daa : Collection(dec())
. 4. rho : Decl
. 5. te : Label→Label→𝔹
. 6. ∀i:ℕ||r.args||. trace_consistent(rho;daa;te;r.args[i])
. 7. ¬subst_mentions_trace(as)
. 8. i : ℕ||map(λt.term_subst2(as;t);r.args)||
. 9. trace_consistent(rho;daa;te;r.args[i])
. 10. ||map(λt.term_subst2(as;t);r.args)|| = ||r.args||
. ⊢  trace_consistent(rho;daa;te;map(λt.term_subst2(as;t);r.args)[i])
. | by RepeatFor 2 (ParallelOp (-2)) THEN ParallelOp (-1)
.
. |    THEN Subst’ map(λt.term_subst2(as;t);r.args)[i]
. |                =
. |                term_subst2(as;r.args[i]) (-1)
. |
. |\
. | 1. r : rel()
. | 2. as : (Label×Term) List
. | 3. daa : Collection(dec())
. | 4. rho : Decl
. | 5. te : Label→Label→𝔹
. | 6. ∀i:ℕ||r.args||. trace_consistent(rho;daa;te;r.args[i])
. | 7. ¬subst_mentions_trace(as)
```

```
. | 8. i : ℕ||map(λt.term_subst2(as;t);r.args)||
. | 9. ∀g:Label.
. |    term_mentions_guard(g;r.args[i])
. |    ⇒
. |    ((Σ[[<d ∈ daa | te(g,d.lbl)>]] rho) List)
. |    ⊆ (rho(lbl_pr(<Trace, g>)))
. | 10. ||map(λt.term_subst2(as;t);r.args)|| = ||r.args||
. | 11. g : Label
. | 12. term_mentions_guard(g;map(λt.term_subst2(as;t);r.args)[i])
. | 13. ∀x,y:Term. x = y ⇒ (x ∼ y)
. | ⊢  map(λt.term_subst2(as;t);r.args)[i] = term_subst2(as;r.args[i])
. |   by RWO "map_select" 0 THEN Reduce 0 THEN Auto
. |
. .\
. . 1. r : rel()
. . 2. as : (Label×Term) List
. . 3. daa : Collection(dec())
. . 4. rho : Decl
. . 5. te : Label→Label→𝔹
. . 6. ∀i:ℕ||r.args||. trace_consistent(rho;daa;te;r.args[i])
. . 7. ¬subst_mentions_trace(as)
. . 8. i : ℕ||map(λt.term_subst2(as;t);r.args)||
. . 9. ∀g:Label.
. .    term_mentions_guard(g;r.args[i])
. .    ⇒
. .    ((Σ[[<d ∈ daa | te(g,d.lbl)>]] rho) List)
. .    ⊆ (rho(lbl_pr(<Trace, g>)))
. . 10. ||map(λt.term_subst2(as;t);r.args)|| = ||r.args||
. . 11. g : Label
. . 12. term_mentions_guard(g;term_subst2(as;r.args[i]))
. . ⊢  term_mentions_guard(g;r.args[i])
. .   by BackThruLemma' `term_subst2_mentions_guard` THEN Auto
END
```