```
trace_consistent_wp2  THM

⊢  ∀A:ioa{i:l}(), Q:Fmla, rho:Decl, R:(Label→Label→𝔹), k:Label.
|  ¬ioa_mentions_trace(A)
|  ⇒
|  trace_consistent_pred(rho;A.da;R;Q)
|  ⇒
|  trace_consistent_pred(rho;A.da;R;wp2(A;k;Q))  by Auto
|
1. A : ioa{i:l}()
2. Q : Fmla
3. rho : Decl
4. R : Label→Label→𝔹
5. k : Label
6. ¬ioa_mentions_trace(A)
7. trace_consistent_pred(rho;A.da;R;Q)
⊢  trace_consistent_pred(rho;A.da;R;wp2(A;k;Q))
| by Assert ∀r:rel().
|    r ∈ Q ⇒ trace_consistent_pred(rho;A.da;R;wp2_rel(A;k;r))
|
|\
| .....assertion .....
| 1. A : ioa{i:l}()
| 2. Q : Fmla
| 3. rho : Decl
| 4. R : Label→Label→𝔹
| 5. k : Label
| 6. ¬ioa_mentions_trace(A)
| 7. trace_consistent_pred(rho;A.da;R;Q)
| ⊢  ∀r:rel().
| |  r ∈ Q ⇒ trace_consistent_pred(rho;A.da;R;wp2_rel(A;k;r))
| | by Auto
| |
| |\
| | 1. A : ioa{i:l}()
| | 2. Q : Fmla
| | 3. rho : Decl
| | 4. R : Label→Label→𝔹
| | 5. k : Label
| | 6. ¬ioa_mentions_trace(A)
| | 7. trace_consistent_pred(rho;A.da;R;Q)
| | 8. r : rel()
| | 9. r ∈ Q
| | ⊢  trace_consistent_pred(rho;A.da;R;wp2_rel(A;k;r))
| |   by BackThruLemma ‘trace_consistent_wp2_rel‘ THEN Auto
| |
| |      THEN EasyHypThen Auto
| |
| |
| .\
| . .....subterm .....
| . 1. A : ioa{i:l}()
| . 2. Q : Fmla
| . 3. rho : Decl
| . 4. R : Label→Label→𝔹
```

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| . 5. Label
| . 6. ¬ioa_mentions_trace(A)
| . 7. trace_consistent_pred(rho;A.da;R;Q)
| . 8. rel()
| . ⊢  Q ∈ Collection(rel())  by Fold ‘pred‘ 0 THEN Auto
|
.\
. 1. A : ioa{i:l}()
. 2. Q : Fmla
. 3. rho : Decl
. 4. R : Label→Label→𝔹
. 5. k : Label
. 6. ¬ioa_mentions_trace(A)
. 7. trace_consistent_pred(rho;A.da;R;Q)
. 8. ∀r:rel().
. |  r ∈ Q ⇒ trace_consistent_pred(rho;A.da;R;wp2_rel(A;k;r))
. ⊢  trace_consistent_pred(rho;A.da;R;wp2(A;k;Q))
. | by ...
. |
. |\
. | 1. A : ioa{i:l}()
. | 2. Q : Fmla
. | 3. rho : Decl
. | 4. R : Label→Label→𝔹
. | 5. k : Label
. | 6. ¬ioa_mentions_trace(A)
. | 7. trace_consistent_pred(rho;A.da;R;Q)
. | 8. ∀r:rel().
. | |  r ∈ Q ⇒ trace_consistent_pred(rho;A.da;R;wp2_rel(A;k;r))
. | 9. r : rel()
. | 10. r ∈ wp2(A;k;Q)
. | ⊢  trace_consistent_rel(rho;A.da;R;r)
. | | by ...
. | |
. | 1. A : ioa{i:l}()
. | 2. Q : Fmla
. | 3. rho : Decl
. | 4. R : Label→Label→𝔹
. | 5. k : Label
. | 6. ¬ioa_mentions_trace(A)
. | 7. trace_consistent_pred(rho;A.da;R;Q)
. | 8. ∀r:rel().
. | |  r ∈ Q ⇒ trace_consistent_pred(rho;A.da;R;wp2_rel(A;k;r))
. | 9. r : rel()
. | 10. r@0 : rel()
. | 11. r@0 ∈ Q
. | 12. r ∈ wp2_rel(A;k;r@0)
. | ⊢  trace_consistent_rel(rho;A.da;R;r)
. | | by InstWithVar (-3) (-5) THENA Auto
. | |
. | 1. A : ioa{i:l}()
. | 2. Q : Fmla
. | 3. rho : Decl
. | 4. R : Label→Label→𝔹
. | 5. k : Label
```

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. | 6. ¬ioa_mentions_trace(A)
. | 7. trace_consistent_pred(rho;A.da;R;Q)
. | 8. ∀r:rel().
. |    r ∈ Q ⇒ trace_consistent_pred(rho;A.da;R;wp2_rel(A;k;r))
. | 9. r : rel()
. | 10. r@0 : rel()
. | 11. r@0 ∈ Q
. | 12. r ∈ wp2_rel(A;k;r@0)
. | 13. trace_consistent_pred(rho;A.da;R;wp2_rel(A;k;r@0))
. | ⊢  trace_consistent_rel(rho;A.da;R;r)
. |   by ...
. | BackThruHyp (-1) THEN Auto
. |
. .\
. . .....subterm .....
. . 1. A : ioa{i:l}()
. . 2. Q : Fmla
. . 3. rho : Decl
. . 4. R : Label→Label→𝔹
. . 5. k : Label
. . 6. ¬ioa_mentions_trace(A)
. . 7. trace_consistent_pred(rho;A.da;R;Q)
. . 8. ∀r:rel().
. .    r ∈ Q ⇒ trace_consistent_pred(rho;A.da;R;wp2_rel(A;k;r))
. . 9. rel()
. . ⊢  wp2(A;k;Q) ∈ Collection(rel())  by Fold ‘pred‘ 0 THEN Auto
END
```