```
wp2_addprime  THM

⊢  ∀A:ioa{i:l}(), a:Label, P:Fmla. wp2(A;a;(P)') = wp(A;a;P)
| by Auto THEN Unfolds ``col_equal wp2 wp`` 0

|    THEN Unfold `smts_eff_pred` 0
THEN Unfold `smts_eff_rel` 0

|    THEN RW ColMemberC 0
|
1. A : ioa{i:l}()
2. a : Label
3. P : Fmla
⊢  ∀x:rel().
|  (∃r:rel().
|   r ∈ (P)'
|   & x ∈ col_subst2(λx.smts_eff(action_effect(a;A.eff;A.frame);x);r))
|  ⟺
|  (∃r:rel().
|   r ∈ P
|   & x ∈ col_subst(λx.smts_eff(action_effect(a;A.eff;A.frame);x);r))
| by Unfold `pred_addprime` 0 THEN
RW ColMemberC 0 THEN

|    RWW "member_col_subst" 0
THENM RWW "member_col_subst2" 0

|    THEN (Auto THEN Try (Fold `pred` 0 THEN Auto)

|               THEN Reduce 0
|           THEN Auto)
|
|\
| 1. A : ioa{i:l}()
| 2. a : Label
| 3. P : Fmla
| 4. x : rel()
| 5. ∃r:rel().
| |  (∃r@0:rel(). r@0 ∈ P & r = (r@0)')
| |  & (∃as:(Label×Term) List.
| |     1of(unzip(as)) = rel_primed_vars(r)
| |     & (∀i:ℕ.
| |        i<||as||
| |        ⇒
| |        2of(as[i]) ∈ (λx.
| |                       smts_eff(action_effect(a;A.eff;A.frame);x))
| |                      (1of(as[i])))
| |     & x = rel_subst2(as;r))
| ⊢  ∃r:rel().
| |  r ∈ P
| |  & (∃as:(Label×Term) List.
| |     1of(unzip(as)) = rel_vars(r)
| |     & (∀i:ℕ.
| |        i<||as||
| |        ⇒
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| |        2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...))
| |     & x = rel_subst(as;r))
| | by ExRepD THEN InstConcl [r@0
| |    ] THENA (Auto THEN Try (Fold ‘pred‘ 0 THEN Auto))
| |
| 1. A : ioa{i:l}()
| 2. a : Label
| 3. P : Fmla
| 4. x : rel()
| 5. r : rel()
| 6. r@0 : rel()
| 7. r@0 ∈ P
| 8. r = (r@0)'
| 9. as : (Label×Term) List
| 10. 1of(unzip(as)) = rel_primed_vars(r)
| 11. ∀i:ℕ.
| |   i<||as||
| |   ⇒
| |   2of(as[i]) ∈ (λx.smts_eff(action_effect(a;A.eff;A.frame);x))
| |                (1of(as[i]))
| 12. x = rel_subst2(as;r)
| ⊢  r@0 ∈ P
| |  & (∃as:(Label×Term) List.
| |     1of(unzip(as)) = rel_vars(r@0)
| |     & (∀i:ℕ.
| |        i<||as||
| |        ⇒
| |        2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...))
| |     & x = rel_subst(as;r@0))
| | by D 0 THEN Try Trivial THEN InstConcl [as] THENA Auto
|
| |    THEN All Reduce THEN D 0 THEN Try Trivial
|
| |    THEN Try (D 0 THEN Try Trivial)
| |
| |\
| | 1. A : ioa{i:l}()
| | 2. a : Label
| | 3. P : Fmla
| | 4. x : rel()
| | 5. r : rel()
| | 6. r@0 : rel()
| | 7. r@0 ∈ P
| | 8. r = (r@0)'
| | 9. as : (Label×Term) List
| | 10. 1of(unzip(as)) = rel_primed_vars(r)
| | 11. ∀i:ℕ.
| | |   i<||as||
| | |   ⇒
| | |   2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...)
| | 12. x = rel_subst2(as;r)
| | ⊢  1of(unzip(as)) = rel_vars(r@0)
| | | by SubstFor 1of(unzip(as)) 0
| | THEN SubstFor r 0
| | |
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```
| | 1. A : ioa{i:l}()
| | 2. a : Label
| | 3. P : Fmla
| | 4. x : rel()
| | 5. r : rel()
| | 6. r@0 : rel()
| | 7. r@0 ∈ P
| | 8. r = (r@0)'
| | 9. as : (Label×Term) List
| | 10. 1of(unzip(as)) = rel_primed_vars(r)
| | 11. ∀i:ℕ.
| |     i<||as||
| |     ⇒
| |     2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...)
| | 12. x = rel_subst2(as;r)
| | ⊢  rel_primed_vars((r@0)') = rel_vars(r@0)
| |   by RW0 "rel_vars_addprime" 0 THEN Auto
| |
| .\
| . 1. A : ioa{i:l}()
| . 2. a : Label
| . 3. P : Fmla
| . 4. x : rel()
| . 5. r : rel()
| . 6. r@0 : rel()
| . 7. r@0 ∈ P
| . 8. r = (r@0)'
| . 9. as : (Label×Term) List
| . 10. 1of(unzip(as)) = rel_primed_vars(r)
| . 11. ∀i:ℕ.
| . |   i<||as||
| . |   ⇒
| . |   2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...)
| . 12. x = rel_subst2(as;r)
| . ⊢  x = rel_subst(as;r@0)
| . | by SubstFor x 0 THEN
| . SubstFor r 0 THEN
| .
| . |    BackThruLemma ‘rel_subst2_addprime‘
| . THEN Auto
| . |
| . 1. A : ioa{i:l}()
| . 2. a : Label
| . 3. P : Fmla
| . 4. x : rel()
| . 5. r : rel()
| . 6. r@0 : rel()
| . 7. r@0 ∈ P
| . 8. r = (r@0)'
| . 9. as : (Label×Term) List
| . 10. 1of(unzip(as)) = rel_primed_vars(r)
| . 11. ∀i:ℕ.
| . |   i<||as||
| . |   ⇒
| . |   2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...)
```

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| . 12. x = rel_subst2(as;r)
| . ⊢  1of(unzip(as)) = rel_vars(r@0)
| . | by Subst rel_vars(r@0) = rel_primed_vars(r) 0 THEN Auto
| . |
| . .....equality .....
| . 1. A : ioa{i:l}()
| . 2. a : Label
| . 3. P : Fmla
| . 4. x : rel()
| . 5. r : rel()
| . 6. r@0 : rel()
| . 7. r@0 ∈ P
| . 8. r = (r@0)'
| . 9. as : (Label×Term) List
| . 10. 1of(unzip(as)) = rel_primed_vars(r)
| . 11. ∀i:ℕ.
| .     i<||as||
| .     ⇒
| .     2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...)
| . 12. x = rel_subst2(as;r)
| . ⊢  rel_vars(r@0) = rel_primed_vars(r)
| .   by SubstFor r 0 THEN RWW "rel_vars_addprime" 0 THEN Auto•
|
.\
. 1. A : ioa{i:l}()
. 2. a : Label
. 3. P : Fmla
. 4. x : rel()
. 5. ∃r:rel().
. |  r ∈ P
. |  & (∃as:(Label×Term) List.
. |     1of(unzip(as)) = rel_vars(r)
. |     & (∀i:ℕ.
. |        i<||as||
. |        ⇒
. |        2of(as[i]) ∈ (λx.
. |                       smts_eff(action_effect(a;A.eff;A.frame);x))
. |                      (1of(as[i])))
. |     & x = rel_subst(as;r))
. ⊢  ∃r:rel().
. |  (∃r@0:rel(). r@0 ∈ P & r = (r@0)')
. |  & (∃as:(Label×Term) List.
. |     1of(unzip(as)) = rel_primed_vars(r)
. |     & (∀i:ℕ.
. |        i<||as||
. |        ⇒
. |        2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...))
. |     & x = rel_subst2(as;r))
. | by ExRepD THEN InstConcl [(r)']
.
. |    THENA (Auto THEN Try (Fold `pred` 0 THEN Auto))
. THEN D 0
.
. |    ...
. THEN InstConcl [as] THENA Auto
```

```
. THEN All Reduce
. THEN D 0
. |
. THEN Try (D 0 THEN Try Trivial)
. |
. |\
. | 1. A : ioa{i:l}()
. | 2. a : Label
. | 3. P : Fmla
. | 4. x : rel()
. | 5. r : rel()
. | 6. r ∈ P
. | 7. as : (Label×Term) List
. | 8. 1of(unzip(as)) = rel_vars(r)
. | 9. ∀i:ℕ.
. | |  i<||as||
. | |  ⇒
. | |  2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...)
. | 10. x = rel_subst(as;r)
. | ⊢  1of(unzip(as)) = rel_primed_vars((r)')
. | | by SubstFor 1of(unzip(as)) 0
. | THEN SubstFor r 0
. | |
. | 1. A : ioa{i:l}()
. | 2. a : Label
. | 3. P : Fmla
. | 4. x : rel()
. | 5. r : rel()
. | 6. r ∈ P
. | 7. as : (Label×Term) List
. | 8. 1of(unzip(as)) = rel_vars(r)
. | 9. ∀i:ℕ.
. |    i<||as||
. |    ⇒
. |    2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...)
. | 10. x = rel_subst(as;r)
. | ⊢  rel_vars(r) = rel_primed_vars((r)')
. |   by RWO "rel_vars_addprime" 0 THEN Auto
. |
. .\
. . 1. A : ioa{i:l}()
. . 2. a : Label
. . 3. P : Fmla
. . 4. x : rel()
. . 5. r : rel()
. . 6. r ∈ P
. . 7. as : (Label×Term) List
. . 8. 1of(unzip(as)) = rel_vars(r)
. . 9. ∀i:ℕ.
. . |  i<||as||
. . |  ⇒
. . |  2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...)
. . 10. x = rel_subst(as;r)
. . ⊢  x = rel_subst2(as;(r)')  by SubstFor x 0 THEN
. . SubstFor r 0
```

```
. . |
. . 1. A : ioa{i:l}()
. . 2. a : Label
. . 3. P : Fmla
. . 4. x : rel()
. . 5. r : rel()
. . 6. r ∈ P
. . 7. as : (Label×Term) List
. . 8. 1of(unzip(as)) = rel_vars(r)
. . 9. ∀i:ℕ.
. .    i<||as||
. .    ⇒
. .    2of(as[i]) ∈ smts_eff(action_effect(a;A.eff;A.frame);...)
. . 10. x = rel_subst(as;r)
. . ⊢  rel_subst(as;r) = rel_subst2(as;(r)')
. .   by Symmetry THEN BackThruLemma `rel_subst2_addprime` THEN Auto
END
```