before-map THM

```
⊢  ∀T,T':Type, f:(T→T'), L:T List, x',y':T'.
|  x' before y' ∈ map(f;L)
|  ⟺
|  (∃x,y:T. x before y ∈ L & f(x) = x' & f(y) = y')
| by InductionOnList THEN Reduce 0
|
|\
| 1. T : Type
| 2. T' : Type
| 3. f : T→T'
| 4. T List
| ⊢  ∀x',y':T'.
|    x' before y' ∈ nil
|    ⟺
|    (∃x,y:T. x before y ∈ nil & f(x) = x' & f(y) = y')
|   by RWO Thm* ∀x,y:T. x before y ∈ nil ⟺ False    {nil_before} 0
|      THEN
|      ExRepD
|
.\
. 1. T : Type
. 2. T' : Type
. 3. f : T→T'
. 4. T List
. 5. u : T
. 6. v : T List
. 7. ∀x',y':T'.
. |  x' before y' ∈ map(f;v)
. |  ⟺
. |  (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. ⊢  ∀x',y':T'.
. |  x' before y' ∈ [(f(u)) / map(f;v)]
. |  ⟺
. |  (∃x,y:T. x before y ∈ [u / v] & f(x) = x' & f(y) = y')
. | by UnivCD
. |    THEN
. |    RWO
. |    Thm* ∀l:T List, a,x,y:T.
. |         x before y ∈ [a / l] ⟺ x = a & (y ∈ l) ∨ x before y ∈ l
. |    {cons_before}
. |    0
. |    THEN
. |    ExRepD
. |
. |\
. | 1. T : Type
. | 2. T' : Type
. | 3. f : T→T'
. | 4. T List
. | 5. u : T
. | 6. v : T List
. | 7. ∀x',y':T'.
. | |  x' before y' ∈ map(f;v)
```

```
. | |  ⟺
. | |  (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. | 8. x' : T'
. | 9. y' : T'
. | 10. x' = f(u) & (y' ∈ map(f;v)) ∨ x' before y' ∈ map(f;v)
. | ⊢  ∃x,y:T.
. | |  x = u & (y ∈ v) ∨ x before y ∈ v & f(x) = x' & f(y) = y'
. | | by SplitOrHyps THEN ExRepD
. | |
. | |\
. | | 1. T : Type
. | | 2. T' : Type
. | | 3. f : T→T'
. | | 4. T List
. | | 5. u : T
. | | 6. v : T List
. | | 7. ∀x',y':T'.
. | | |  x' before y' ∈ map(f;v)
. | | |  ⟺
. | | |  (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. | | 8. x' : T'
. | | 9. y' : T'
. | | 10. x' = f(u)
. | | 11. (y' ∈ map(f;v))
. | | ⊢  ∃x,y:T.
. | | |  x = u & (y ∈ v) ∨ x before y ∈ v & f(x) = x' & f(y) = y'
. | | | by RWO
. | | |    Thm* ∀a:T List, x:T', f:(T→T').
. | | |         (x ∈ map(f;a)) ⟺ (∃y:T. (y ∈ a) & x = f(y))
. | | |    {member_map}
. | | |    -1
. | | |    THEN
. | | |    ExRepD
. | | |
. | | 1. T : Type
. | | 2. T' : Type
. | | 3. f : T→T'
. | | 4. T List
. | | 5. u : T
. | | 6. v : T List
. | | 7. ∀x',y':T'.
. | |    x' before y' ∈ map(f;v)
. | |    ⟺
. | |    (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. | | 8. x' : T'
. | | 9. y' : T'
. | | 10. x' = f(u)
. | | 11. y : T
. | | 12. (y ∈ v)
. | | 13. y' = f(y)
. | | ⊢  ∃x,y:T.
. | |    x = u & (y ∈ v) ∨ x before y ∈ v & f(x) = x' & f(y) = y'
. | |   by InstConcl [u;y] THEN OrLeft
. | |
. | .\
```

```
. | . 1. T : Type
. | . 2. T' : Type
. | . 3. f : T→T'
. | . 4. T List
. | . 5. u : T
. | . 6. v : T List
. | . 7. ∀x',y':T'.
. | . |  x' before y' ∈ map(f;v)
. | . |  ⟺
. | . |  (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. | . 8. x' : T'
. | . 9. y' : T'
. | . 10. x' before y' ∈ map(f;v)
. | . ⊢  ∃x,y:T.
. | . |  x = u & (y ∈ v) ∨ x before y ∈ v & f(x) = x' & f(y) = y'
. | . | by InstHyp [x';y'] -4 THEN ThinTrivial
. | . |    THEN
. | . |    RepeatFor 2 (ParallelOp -1)
. | . |
. | . 1. T : Type
. | . 2. T' : Type
. | . 3. f : T→T'
. | . 4. T List
. | . 5. u : T
. | . 6. v : T List
. | . 7. ∀x',y':T'.
. | .    x' before y' ∈ map(f;v)
. | .    ⟺
. | .    (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. | . 8. x' : T'
. | . 9. y' : T'
. | . 10. x' before y' ∈ map(f;v)
. | . 11. x' before y' ∈ map(f;v) ⇐ (∃x,y:T.
. | .     x before y ∈ v & f(x) = x' & f(y) = y')
. | . 12. x : T
. | . 13. y : T
. | . 14. x before y ∈ v & f(x) = x' & f(y) = y'
. | . ⊢  x = u & (y ∈ v) ∨ x before y ∈ v & f(x) = x' & f(y) = y'
. | .   by Auto THEN OrRight
. |
. .\
. . 1. T : Type
. . 2. T' : Type
. . 3. f : T→T'
. . 4. T List
. . 5. u : T
. . 6. v : T List
. . 7. ∀x',y':T'.
. . |  x' before y' ∈ map(f;v)
. . |  ⟺
. . |  (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. . 8. x' : T'
. . 9. y' : T'
. . 10. x : T
. . 11. y : T
```

```
. . 12. x = u & (y ∈ v) ∨ x before y ∈ v
. . 13. f(x) = x'
. . 14. f(y) = y'
. . ⊢  x' = f(u) & (y' ∈ map(f;v)) ∨ x' before y' ∈ map(f;v)
. . | by ParallelOp -3
. . |
. . |\
. . | 1. T : Type
. . | 2. T' : Type
. . | 3. f : T→T'
. . | 4. T List
. . | 5. u : T
. . | 6. v : T List
. . | 7. ∀x',y':T'.
. . |    x' before y' ∈ map(f;v)
. . |    ⟺
. . |    (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. . | 8. x' : T'
. . | 9. y' : T'
. . | 10. x : T
. . | 11. y : T
. . | 12. x = u
. . | 13. (y ∈ v)
. . | 14. f(x) = x'
. . | 15. f(y) = y'
. . | ⊢  x' = f(u)  by RevHypSubst -4 0
. . |
. . |\
. . | 1. T : Type
. . | 2. T' : Type
. . | 3. f : T→T'
. . | 4. T List
. . | 5. u : T
. . | 6. v : T List
. . | 7. ∀x',y':T'.
. . |    x' before y' ∈ map(f;v)
. . |    ⟺
. . |    (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. . | 8. x' : T'
. . | 9. y' : T'
. . | 10. x : T
. . | 11. y : T
. . | 12. x = u
. . | 13. (y ∈ v)
. . | 14. f(x) = x'
. . | 15. f(y) = y'
. . | ⊢  (y' ∈ map(f;v))
. . |   by RWO
. . |      Thm* ∀a:T List, x:T', f:(T→T').
. . |           (x ∈ map(f;a)) ⟺ (∃y:T. (y ∈ a) & x = f(y))
. . |      {member_map}
. . |      0
. . |      THEN
. . |      InstConcl [y]
. . |
```

```
. . .\
. . . 1. T : Type
. . . 2. T' : Type
. . . 3. f : T→T'
. . . 4. T List
. . . 5. T
. . . 6. v : T List
. . . 7. ∀x',y':T'.
. . . |  x' before y' ∈ map(f;v)
. . . |  ⟺
. . . |  (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. . . 8. x' : T'
. . . 9. y' : T'
. . . 10. x : T
. . . 11. y : T
. . . 12. x before y ∈ v
. . . 13. f(x) = x'
. . . 14. f(y) = y'
. . . ⊢  x' before y' ∈ map(f;v)  by BackThruSomeHyp
. . . |
. . . 1. T : Type
. . . 2. T' : Type
. . . 3. f : T→T'
. . . 4. T List
. . . 5. T
. . . 6. v : T List
. . . 7. ∀x',y':T'.
. . .    x' before y' ∈ map(f;v)
. . .    ⟺
. . .    (∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y')
. . . 8. x' : T'
. . . 9. y' : T'
. . . 10. x : T
. . . 11. y : T
. . . 12. x before y ∈ v
. . . 13. f(x) = x'
. . . 14. f(y) = y'
. . . ⊢  ∃x,y:T. x before y ∈ v & f(x) = x' & f(y) = y'
. . .   by InstConcl [x;y]
END
```