```
finite-decidable-set  THM

⊢  ∀T:Type, P:(T→Prop).
|  (∀x:T. Dec(P(x)))
|  ⇒
|  (finite-type({x:T| P(x) }) ⟺ (∃L:T List. ∀x:T. P(x) ⇒ (x ∈ L)))
| by UnivCD
|    THEN
|    RW0
|    Thm* ∀P:(T→Prop).
|         (∀x:T. SqStable(P(x)))
|         ⇒
|         (finite-type({x:T| P(x) })
|          ⟺
|          (∃L:T List. ∀x:T. P(x) ⟺ (x ∈ L)))
|    {finite_DASH_set_DASH_type}
|    0
|    THEN
|    RepeatFor 2 (Analyze 0)
|
|\
| 1. T : Type
| 2. P : T→Prop
| 3. ∀x:T. Dec(P(x))
| 4. ∃L:T List. ∀x:T. P(x) ⟺ (x ∈ L)
| ⊢  ∃L:T List. ∀x:T. P(x) ⇒ (x ∈ L)
|   by RepeatFor 2 (ParallelOp -1) THEN BackThruSomeHyp
|
.\
. 1. T : Type
. 2. P : T→Prop
. 3. ∀x:T. Dec(P(x))
. 4. ∃L:T List. ∀x:T. P(x) ⇒ (x ∈ L)
. ⊢  ∃L:T List. ∀x:T. P(x) ⟺ (x ∈ L)  by ExRepD
. |
. 1. T : Type
. 2. P : T→Prop
. 3. ∀x:T. Dec(P(x))
. 4. L : T List
. 5. ∀x:T. P(x) ⇒ (x ∈ L)
. ⊢  ∃L:T List. ∀x:T. P(x) ⟺ (x ∈ L)
. | by Assert (∃f:(T→𝔹). ∀x:T. f(x) ⟺ P(x))
. |
. |\
. | .....assertion .....
. | 1. T : Type
. | 2. P : T→Prop
. | 3. ∀x:T. Dec(P(x))
. | 4. L : T List
. | 5. ∀x:T. P(x) ⇒ (x ∈ L)
. | ⊢  ∃f:(T→𝔹). ∀x:T. f(x) ⟺ P(x)
. | | by Thin -1 THEN Unfolds [‘all‘;‘decidable‘] 3
. | |    THEN
. | |    RenameVar ‘f’ 3
. | |    THEN
```

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. | |    InstConcl [λx.InjCase(f(x); a. true₂, false₂)]
. | |    THEN
. | |    Reduce 0
. | |    THEN
. | |    Analyze 0
. | |
. | 1. T : Type
. | 2. P : T→Prop
. | 3. f : x:T→P(x) ∨ ¬P(x)
. | 4. T List
. | 5. x : T
. | ⊢  InjCase(f(x); a. true₂, false₂) ⟺ P(x)
. | | by GenConclAtAddr [1;1;1] THEN Analyze -2 THEN Reduce 0
. | |
. | 1. T : Type
. | 2. P : T→Prop
. | 3. f : x:T→P(x) ∨ ¬P(x)
. | 4. T List
. | 5. x : T
. | 6. y : ¬P(x)
. | 7. f(x) = inr(y)
. | 8. P(x)
. | ⊢  False  by Analyze -3
. |
. .\
. . 1. T : Type
. . 2. P : T→Prop
. . 3. ∀x:T. Dec(P(x))
. . 4. L : T List
. . 5. ∀x:T. P(x) ⇒ (x ∈ L)
. . 6. ∃f:(T→𝔹). ∀x:T. f(x) ⟺ P(x)
. . ⊢  ∃L:T List. ∀x:T. P(x) ⟺ (x ∈ L)
. .   by ExRepD THEN InstConcl [filter(f;L)]
. .      THEN
. .      RWO
. .      Thm* ∀P:(T→𝔹), L:T List, x:T.
. .           (x ∈ filter(P;L)) ⟺ (x ∈ L) & P(x)
. .      {member_filter}
. .      0
. .      THEN
. .      BackThruSomeHyp
END
```