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firstn_map  THM

⊢  ∀f:(Top→Top), n:ℕ, l:Top List.
|  firstn(n;map(f;l)) ∼ map(f;firstn(n;l))  by InductionOnNat
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| .....basecase .....
| 1. f : Top→Top
| ⊢  ∀l:Top List. firstn(0;map(f;l)) ∼ map(f;firstn(0;l))
|   by (UnivCD
|       THEN
|       (RWO Thm* ∀L:Top List. firstn(0;L) ∼ nil    {first0} 0))
|      THEN
|      Reduce 0
|
.\
. .....upcase .....
. 1. f : Top→Top
. 2. n : ℤ
. 3. 0<n
. 4. ∀l:Top List. firstn(n-1;map(f;l)) ∼ map(f;firstn(n-1;l))
. ⊢  ∀l:Top List. firstn(n;map(f;l)) ∼ map(f;firstn(n;l))
. | by InductionOnList THEN Reduce 0
. |
. |\
. | 1. f : Top→Top
. | 2. n : ℤ
. | 3. 0<n
. | 4. ∀l:Top List. firstn(n-1;map(f;l)) ∼ map(f;firstn(n-1;l))
. | 5. Top List
. | ⊢  nil ∼ nil  by Trivial
. |
. .\
. . 1. f : Top→Top
. . 2. n : ℤ
. . 3. 0<n
. . 4. ∀l:Top List. firstn(n-1;map(f;l)) ∼ map(f;firstn(n-1;l))
. . 5. Top List
. . 6. u : Top
. . 7. v : Top List
. . 8. firstn(n;map(f;v)) ∼ map(f;firstn(n;v))
. . ⊢  firstn(n;[(f(u)) / map(f;v)]) ∼ map(f;firstn(n;[u / v]))
. . | by RecUnfold ‘firstn‘ 0 THEN Reduce 0 THEN SplitOnConclITE
. . |
. . |\
. . | .....truecase .....
. . | 1. f : Top→Top
. . | 2. n : ℤ
. . | 3. 0<n
. . | 4. ∀l:Top List. firstn(n-1;map(f;l)) ∼ map(f;firstn(n-1;l))
. . | 5. Top List
. . | 6. u : Top
. . | 7. v : Top List
. . | 8. firstn(n;map(f;v)) ∼ map(f;firstn(n;v))
. . | 9. 0<n
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. . | ⊢  [(f(u)) / firstn(n-1;map(f;v))] ∼ map(f;[u / firstn(n-1;v)])
. . | | by Reduce 0 THEN Analyze THEN Try Trivial
. . | |
. . | 1. f : Top→Top
. . | 2. n : ℤ
. . | 3. 0<n
. . | 4. ∀l:Top List. firstn(n-1;map(f;l)) ∼ map(f;firstn(n-1;l))
. . | 5. Top List
. . | 6. Top
. . | 7. v : Top List
. . | 8. firstn(n;map(f;v)) ∼ map(f;firstn(n;v))
. . | 9. 0<n
. . | ⊢  firstn(n-1;map(f;v)) ∼ map(f;firstn(n-1;v))  by BackThru 4
. . |
. . .\
. . . .....falsecase .....
. . . 1. f : Top→Top
. . . 2. n : ℤ
. . . 3. 0<n
. . . 4. ∀l:Top List. firstn(n-1;map(f;l)) ∼ map(f;firstn(n-1;l))
. . . 5. Top List
. . . 6. Top
. . . 7. v : Top List
. . . 8. firstn(n;map(f;v)) ∼ map(f;firstn(n;v))
. . . 9. n≤0
. . . ⊢  nil ∼ map(f;nil)  by Auto
END
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