```
firstn_upto  THM

⊢  ∀n,m:ℕ. firstn(n;upto(m)) ~ if n≤₂m→ upto(n) else upto(m) fi
| by UnivCD THEN SplitOnConclITE
|
|\
| .....truecase .....
| 1. n : ℕ
| 2. m : ℕ
| 3. n≤m
| ⊢  firstn(n;upto(m)) ~ upto(n)
| | by Subst (upto(m) ~ (upto(n) @ map(λx.x+n;upto(m-n)))) 0
| |
| |\
| | .....equality .....
| | 1. n : ℕ
| | 2. m : ℕ
| | 3. n≤m
| | ⊢  upto(m) ~ (upto(n) @ map(λx.x+n;upto(m-n)))
| |   by BackThru
| |      Thm* ∀m:ℕ, n:ℕ(m+1).
| |           upto(m) ~ (upto(n) @ map(λx.x+n;upto(m-n)))
| |      {upto_decomp}
| |
| .\
| . 1. n : ℕ
| . 2. m : ℕ
| . 3. n≤m
| . ⊢  firstn(n;upto(n) @ map(λx.x+n;upto(m-n))) ~ upto(n)
| . | by RWO
| . |    Thm* ∀L1,L2:Top List, n:ℕ(||L1||+1).
| . |         firstn(n;L1 @ L2) ~ firstn(n;L1)
| . |    {firstn_append}
| . |    0
| . |
| . |\
| . | .....wf .....
| . | 1. n : ℕ
| . | 2. m : ℕ
| . | 3. n≤m
| . | ⊢  n ∈ ℕ(||upto(n)||+1)
| . |   by RWO Thm* ∀n:ℕ. ||upto(n)|| ~ n    {length_upto} 0
| . |
| . .\
| . . 1. n : ℕ
| . . 2. m : ℕ
| . . 3. n≤m
| . . ⊢  firstn(n;upto(n)) ~ upto(n)
| . .   by RWO
| . .      Thm* ∀L:Top List, n:ℤ. ||L||≤n ⇒ (firstn(n;L) ~ L)
| . .      {firstn_all}
| . .      0
| . .      THEN
| . .      RWO Thm* ∀n:ℕ. ||upto(n)|| ~ n    {length_upto} 0
|
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. .....falsecase .....
. 1. n : ℕ
. 2. m : ℕ
. 3. m<n
. ⊢  firstn(n;upto(m)) ~ upto(m)
.   by RWO
.      Thm* ∀L:Top List, n:ℤ. ||L||≤n ⇒ (firstn(n;L) ~ L)
.      {firstn_all}
.      0
.      THEN
.      RWO Thm* ∀n:ℕ. ||upto(n)|| ~ n    {length_upto} 0
END
```