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fun_exp_add_sq  THM

⊢  ∀n,m:ℕ, f,i:Top. (f^n+m(i)) ~ (f^n(f^m(i)))  by InductionOnNat
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|\
| .....basecase .....
| ⊢  ∀m:ℕ, f,i:Top. (f^0+m(i)) ~ (f^0(f^m(i)))
|   by Analyze 0 THEN Subst ((0+m) ~ m) 0 THEN Reduce 0
|
.\
. .....upcase .....
. 1. n : ℤ
. 2. 0<n
. 3. ∀m:ℕ, f,i:Top. (f^n-1+m(i)) ~ (f^n-1(f^m(i)))
. ⊢  ∀m:ℕ, f,i:Top. (f^n+m(i)) ~ (f^n(f^m(i)))  by UnivCD
. |
. 1. n : ℤ
. 2. 0<n
. 3. ∀m:ℕ, f,i:Top. (f^n-1+m(i)) ~ (f^n-1(f^m(i)))
. 4. m : ℕ
. 5. f : Top
. 6. i : Top
. ⊢  (f^n+m(i)) ~ (f^n(f^m(i)))
. | by Subst (n ~ (n-1+1)) 0 THENL [Auto;Id]
. |    THEN
. |    Subst ((n-1+1+m) ~ (n-1+m+1)) 0
. |    THENL
. |    [Auto;Id]
. |
. 1. n : ℤ
. 2. 0<n
. 3. ∀m:ℕ, f,i:Top. (f^n-1+m(i)) ~ (f^n-1(f^m(i)))
. 4. m : ℕ
. 5. f : Top
. 6. i : Top
. ⊢  (f^n-1+m+1(i)) ~ (f^n-1+1(f^m(i)))
. | by Assert (∀n:ℕ, x:Top. (f^n+1(x)) ~ (f(f^n(x))))
. |
. |\
. | .....assertion .....
. | 1. n : ℤ
. | 2. 0<n
. | 3. ∀m:ℕ, f,i:Top. (f^n-1+m(i)) ~ (f^n-1(f^m(i)))
. | 4. ℕ
. | 5. f : Top
. | 6. Top
. | ⊢  ∀n:ℕ, x:Top. (f^n+1(x)) ~ (f(f^n(x)))  by All Thin
. | |
. | 1. f : Top
. | ⊢  ∀n:ℕ, x:Top. (f^n+1(x)) ~ (f(f^n(x)))
. |   by UnivCD THEN Unfold ‘fun_exp‘ 0
. |      THEN
. |      RW (AddrC [1] (RecUnfoldC ‘primrec‘)) 0
. |      THEN
. |      SplitOnConclITE
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. |      THENL
. |      [Auto;Id]
. |      THEN
. |      Subst ((n+1-1) ~ n) 0
. |      THENL
. |      [Auto;Reduce 0]
. |      THEN
. |      Trivial
. |
. .\
. . 1. n : ℤ
. . 2. 0<n
. . 3. ∀m:ℕ, f,i:Top. (f^n-1+m(i)) ~ (f^n-1(f^m(i)))
. . 4. m : ℕ
. . 5. f : Top
. . 6. i : Top
. . 7. ∀n:ℕ, x:Top. (f^n+1(x)) ~ (f(f^n(x)))
. . ⊢  (f^n-1+m+1(i)) ~ (f^n-1+1(f^m(i)))
. . | by RW (SweepDnC (HypC -1)) 0
. . |
. . 1. n : ℤ
. . 2. 0<n
. . 3. ∀m:ℕ, f,i:Top. (f^n-1+m(i)) ~ (f^n-1(f^m(i)))
. . 4. m : ℕ
. . 5. f : Top
. . 6. i : Top
. . 7. ∀n:ℕ, x:Top. (f^n+1(x)) ~ (f(f^n(x)))
. . ⊢  (f(f^n-1+m(i))) ~ (f(f^n-1(f^m(i))))
. .   by RW (SweepDnC (HypC 3)) 0
END
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