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maximal-in-list  THM

⊢  ∀A:Type, f:(A→ℤ), L:A List. 0<||L|| ⇒ (∃a∈L.(∀x∈L.f(x)≤f(a)))
| by InductionOnList THEN Reduce 0
|
1. A : Type
2. f : A→ℤ
3. A List
4. u : A
5. v : A List
6. 0<||v|| ⇒ (∃a∈v.(∀x∈v.f(x)≤f(a)))
7. 0<||v||+1
⊢  (∃a∈[u / v].(∀x∈[u / v].f(x)≤f(a)))  by DVar ‘v’
|
|\
| 1. A : Type
| 2. f : A→ℤ
| 3. A List
| 4. u : A
| 5. 0<||nil|| ⇒ (∃a∈nil.(∀x∈nil.f(x)≤f(a)))
| 6. 0<||nil||+1
| ⊢  (∃a∈[u].(∀x∈[u].f(x)≤f(a)))
| | by Unfold ‘l_exists‘ 0 THEN InstConcl [u]
| |
| |\
| | 1. A : Type
| | 2. f : A→ℤ
| | 3. A List
| | 4. u : A
| | 5. 0<||nil|| ⇒ (∃a∈nil.(∀x∈nil.f(x)≤f(a)))
| | 6. 0<||nil||+1
| | ⊢  (u ∈ [u])
| |   by RWO Thm* ∀a,x:T. (x ∈ [a]) ⟺ x = a    {member_singleton} 0
| |
| .\
| . 1. A : Type
| . 2. f : A→ℤ
| . 3. A List
| . 4. u : A
| . 5. 0<||nil|| ⇒ (∃a∈nil.(∀x∈nil.f(x)≤f(a)))
| . 6. 0<||nil||+1
| . ⊢  (∀x∈[u].f(x)≤f(u))
| .   by RWO
| .      Thm* ∀P:(T→Prop), x:T, L:T List.
| .           (∀y∈[x / L].P(y)) ⟺ P(x) & (∀y∈L.P(y))
| .      {l_all_cons}
| .      0
| .      THEN
| .      BackThru Thm* ∀P:(T→Prop). (∀x∈nil.P(x))    {l_all_nil}
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. 1. A : Type
. 2. f : A→ℤ
. 3. A List
. 4. u : A
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. 5. u1 : A
. 6. v1 : A List
. 7. 0<||[u1 / v1]|| ⇒ (∃a∈[u1 / v1].(∀x∈[u1 / v1].f(x)≤f(a)))
. 8. 0<||[u1 / v1]||+1
. ⊢  (∃a∈[u; u1 / v1].(∀x∈[u; u1 / v1].f(x)≤f(a)))
. | by Analyze -2
. |    THEN
. |    RWO
. |    Thm* ∀P:(T→Prop), x:T, L:T List.
. |         (∃y∈[x / L].P(y)) ⟺ P(x) ∨ (∃y∈L.P(y))
. |    {l_exists_cons}
. |    0
. |    THEN
. |    DupHyp -1
. |    THEN
. |    Analyze -1
. |    THEN
. |    Decide (f(a)≤f(u))
. |    THENL
. |    [OrLeft;OrRight]
. |
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. | 1. A : Type
. | 2. f : A→ℤ
. | 3. A List
. | 4. u : A
. | 5. u1 : A
. | 6. v1 : A List
. | 7. 0<||[u1 / v1]||+1
. | 8. (∃a∈[u1 / v1].(∀x∈[u1 / v1].f(x)≤f(a)))
. | 9. a : A
. | 10. (a ∈ [u1 / v1])
. | 11. (∀x∈[u1 / v1].f(x)≤f(a))
. | 12. f(a)≤f(u)
. | ⊢  (∀x∈[u; u1 / v1].f(x)≤f(u))
. | | by RWO
. | |    Thm* ∀P:(T→Prop), x:T, L:T List.
. | |         (∀y∈[x / L].P(y)) ⟺ P(x) & (∀y∈L.P(y))
. | |    {l_all_cons}
. | |    0
. | |
. | 1. A : Type
. | 2. f : A→ℤ
. | 3. A List
. | 4. u : A
. | 5. u1 : A
. | 6. v1 : A List
. | 7. 0<||[u1 / v1]||+1
. | 8. (∃a∈[u1 / v1].(∀x∈[u1 / v1].f(x)≤f(a)))
. | 9. a : A
. | 10. (a ∈ [u1 / v1])
. | 11. (∀x∈[u1 / v1].f(x)≤f(a))
. | 12. f(a)≤f(u)
. | ⊢  (∀x∈[u1 / v1].f(x)≤f(u))
. |   by RepeatFor 2 (ParallelOp -2) THEN ParallelOp -1
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. . 1. A : Type
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. . 8. (∃a∈[u1 / v1].(∀x∈[u1 / v1].f(x)≤f(a)))
. . 9. a : A
. . 10. (a ∈ [u1 / v1])
. . 11. (∀x∈[u1 / v1].f(x)≤f(a))
. . 12. ¬f(a)≤f(u)
. . ⊢  (∃a∈[u1 / v1].(∀x∈[u; u1 / v1].f(x)≤f(a)))
. . | by Unfold ‘l_exists‘ 0 THEN InstConcl [a]
. . |
. . 1. A : Type
. . 2. f : A→ℤ
. . 3. A List
. . 4. u : A
. . 5. u1 : A
. . 6. v1 : A List
. . 7. 0<||[u1 / v1]||+1
. . 8. (∃a∈[u1 / v1].(∀x∈[u1 / v1].f(x)≤f(a)))
. . 9. a : A
. . 10. (a ∈ [u1 / v1])
. . 11. (∀x∈[u1 / v1].f(x)≤f(a))
. . 12. ¬f(a)≤f(u)
. . ⊢  (∀x∈[u; u1 / v1].f(x)≤f(a))
. .   by RWO
. .      Thm* ∀P:(T→Prop), x:T, L:T List.
. .           (∀y∈[x / L].P(y)) ⟺ P(x) & (∀y∈L.P(y))
. .      {l_all_cons}
. .      0
END
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