```
member-concat  THM

⊢  ∀T:Type, ll:(T List) List, x:T.
|  (x ∈ concat(ll)) ⟺ (∃l:T List. (l ∈ ll) & (x ∈ l))
| by ((InductionOnList THEN Unfold ‘concat‘ 0 THEN Reduce 0
|      THEN
|      Try (Fold ‘concat‘ 0))
|     THEN
|     (RWW
|      Thm* ∀x:T, l1,l2:T List. (x ∈ l1 @ l2) ⟺ (x ∈ l1) ∨ (x ∈ l2)
|      {member_append}
|      0)
|     THEN
|     (RWW
|      Thm* ∀l:T List, a,x:T. (x ∈ [a / l]) ⟺ x = a ∨ (x ∈ l)
|      {cons_member}
|      0))
|    THEN
|    RWW Thm* ∀x:T. (x ∈ nil) ⟺ False    {nil_member} 0
|    THEN
|    ExRepD
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|\
| 1. T : Type
| 2. (T List) List
| 3. u : T List
| 4. v : (T List) List
| 5. ∀x:T. (x ∈ concat(v)) ⟺ (∃l:T List. (l ∈ v) & (x ∈ l))
| 6. x : T
| 7. (x ∈ u) ∨ (x ∈ concat(v))
| ⊢  ∃l:T List. l = u ∨ (l ∈ v) & (x ∈ l)  by Analyze -1
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| |\
| | 1. T : Type
| | 2. (T List) List
| | 3. u : T List
| | 4. v : (T List) List
| | 5. ∀x:T. (x ∈ concat(v)) ⟺ (∃l:T List. (l ∈ v) & (x ∈ l))
| | 6. x : T
| | 7. (x ∈ u)
| | ⊢  ∃l:T List. l = u ∨ (l ∈ v) & (x ∈ l)
| |   by InstConcl [u] THEN Obvious
| |
| .\
| . 1. T : Type
| . 2. (T List) List
| . 3. u : T List
| . 4. v : (T List) List
| . 5. ∀x:T. (x ∈ concat(v)) ⟺ (∃l:T List. (l ∈ v) & (x ∈ l))
| . 6. x : T
| . 7. (x ∈ concat(v))
| . ⊢  ∃l:T List. l = u ∨ (l ∈ v) & (x ∈ l)
| .   by FwdThru -3 [-1] THEN Obvious
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. 1. T : Type
. 2. (T List) List
. 3. u : T List
. 4. v : (T List) List
. 5. ∀x:T. (x ∈ concat(v)) ⟺ (∃l:T List. (l ∈ v) & (x ∈ l))
. 6. x : T
. 7. l : T List
. 8. l = u ∨ (l ∈ v)
. 9. (x ∈ l)
. ⊢  (x ∈ u) ∨ (x ∈ concat(v))  by Analyze -2
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. | 1. T : Type
. | 2. (T List) List
. | 3. u : T List
. | 4. v : (T List) List
. | 5. ∀x:T. (x ∈ concat(v)) ⟺ (∃l:T List. (l ∈ v) & (x ∈ l))
. | 6. x : T
. | 7. l : T List
. | 8. l = u
. | 9. (x ∈ l)
. | ⊢  (x ∈ u) ∨ (x ∈ concat(v))  by Obvious
. |
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. . 1. T : Type
. . 2. (T List) List
. . 3. u : T List
. . 4. v : (T List) List
. . 5. ∀x:T. (x ∈ concat(v)) ⟺ (∃l:T List. (l ∈ v) & (x ∈ l))
. . 6. x : T
. . 7. l : T List
. . 8. (l ∈ v)
. . 9. (x ∈ l)
. . ⊢  (x ∈ u) ∨ (x ∈ concat(v))  by OrRight THEN BackThruSomeHyp
. . |
. . 1. T : Type
. . 2. (T List) List
. . 3. T List
. . 4. v : (T List) List
. . 5. ∀x:T. (x ∈ concat(v)) ⟺ (∃l:T List. (l ∈ v) & (x ∈ l))
. . 6. x : T
. . 7. l : T List
. . 8. (l ∈ v)
. . 9. (x ∈ l)
. . ⊢  ∃l:T List. (l ∈ v) & (x ∈ l)  by Obvious
END
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