```
mu_wf  THM

⊢  ∀f:(ℕ→𝔹). (∃n:ℕ. f(n)) ⇒ mu(f) ∈ ℕ
| by Assert (∀b:ℕ, f:(ℕ→𝔹). (∃n:ℕ. n≤b & f(n)) ⇒ mu(f) ∈ ℕ)
|
|\
| .....assertion .....
| ⊢  ∀b:ℕ, f:(ℕ→𝔹). (∃n:ℕ. n≤b & f(n)) ⇒ mu(f) ∈ ℕ
| | by InductionOnNat THEN RepeatFor 2 (Analyze 0) THEN ExRepD
| |    THEN
| |    RecUnfold ‘mu‘ 0
| |    THEN
| |    SplitOnConclITE
| |
| |\
| | .....falsecase .....
| | 1. f : ℕ→𝔹
| | 2. n : ℕ
| | 3. n≤0
| | 4. f(n)
| | 5. ¬f(0)
| | ⊢  mu(λx.f(x+1))+1 ∈ ℕ  by Subst’ (n = 0) -2
| |
| .\
| . .....falsecase .....
| . 1. b : ℤ
| . 2. 0<b
| . 3. ∀f:(ℕ→𝔹). (∃n:ℕ. n≤b-1 & f(n)) ⇒ mu(f) ∈ ℕ
| . 4. f : ℕ→𝔹
| . 5. n : ℕ
| . 6. n≤b
| . 7. f(n)
| . 8. ¬f(0)
| . ⊢  mu(λx.f(x+1))+1 ∈ ℕ  by CaseNat 0 ‘n’
| . |
| . |\
| . | 1. b : ℤ
| . | 2. 0<b
| . | 3. ∀f:(ℕ→𝔹). (∃n:ℕ. n≤b-1 & f(n)) ⇒ mu(f) ∈ ℕ
| . | 4. f : ℕ→𝔹
| . | 5. n : ℕ
| . | 6. n≤b
| . | 7. f(n)
| . | 8. ¬f(0)
| . | 9. n = 0
| . | ⊢  mu(λx.f(x+1))+1 ∈ ℕ  by HypSubst’ -1 -3
| . |
| . .\
| . . 1. b : ℤ
| . . 2. 0<b
| . . 3. ∀f:(ℕ→𝔹). (∃n:ℕ. n≤b-1 & f(n)) ⇒ mu(f) ∈ ℕ
| . . 4. f : ℕ→𝔹
| . . 5. n : ℕ
| . . 6. n≤b
| . . 7. f(n)
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| . . 8. ¬f(0)
| . . 9. ¬n = 0
| . . ⊢  mu(λx.f(x+1))+1 ∈ ℕ  by Assert (mu(λx.f(x+1)) ∈ ℕ)
| . . |
| . . |\
| . . | .....assertion .....
| . . | 1. b : ℤ
| . . | 2. 0<b
| . . | 3. ∀f:(ℕ→𝔹). (∃n:ℕ. n≤b-1 & f(n)) ⇒ mu(f) ∈ ℕ
| . . | 4. f : ℕ→𝔹
| . . | 5. n : ℕ
| . . | 6. n≤b
| . . | 7. f(n)
| . . | 8. ¬f(0)
| . . | 9. ¬n = 0
| . . | ⊢  mu(λx.f(x+1)) ∈ ℕ
| . . | | by BackThru 3 THEN Reduce 0 THEN InstConcl [n-1]
| . . | |
| . . | 1. b : ℤ
| . . | 2. 0<b
| . . | 3. ∀f:(ℕ→𝔹). (∃n:ℕ. n≤b-1 & f(n)) ⇒ mu(f) ∈ ℕ
| . . | 4. f : ℕ→𝔹
| . . | 5. n : ℕ
| . . | 6. n≤b
| . . | 7. f(n)
| . . | 8. ¬f(0)
| . . | 9. ¬n = 0
| . . | ⊢  f(n-1+1)  by ArithSimp 0
| . . |
| . . .\
| . . . 1. b : ℤ
| . . . 2. 0<b
| . . . 3. ∀f:(ℕ→𝔹). (∃n:ℕ. n≤b-1 & f(n)) ⇒ mu(f) ∈ ℕ
| . . . 4. f : ℕ→𝔹
| . . . 5. n : ℕ
| . . . 6. n≤b
| . . . 7. f(n)
| . . . 8. ¬f(0)
| . . . 9. ¬n = 0
| . . . 10. mu(λx.f(x+1)) ∈ ℕ
| . . . ⊢  mu(λx.f(x+1))+1 ∈ ℕ  by Auto THEN Analyze -1
|
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. 1. ∀b:ℕ, f:(ℕ→𝔹). (∃n:ℕ. n≤b & f(n)) ⇒ mu(f) ∈ ℕ
. ⊢  ∀f:(ℕ→𝔹). (∃n:ℕ. f(n)) ⇒ mu(f) ∈ ℕ
. | by RepeatFor 2 (Analyze 0) THEN ExRepD
. |
. 1. ∀b:ℕ, f:(ℕ→𝔹). (∃n:ℕ. n≤b & f(n)) ⇒ mu(f) ∈ ℕ
. 2. f : ℕ→𝔹
. 3. n : ℕ
. 4. f(n)
. ⊢  mu(f) ∈ ℕ  by InstHyp [n;f] 1
. |
. .....antecedent .....
. 1. ∀b:ℕ, f:(ℕ→𝔹). (∃n:ℕ. n≤b & f(n)) ⇒ mu(f) ∈ ℕ
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. 2. f : ℕ→𝔹
. 3. n : ℕ
. 4. f(n)
. ⊢  ∃n@0:ℕ. n@0≤n & f(n@0)  by InstConcl [n]
END
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