```
pairwise-cons  THM

⊢  ∀T:Type, x:T, L:T List, P:(T→T→Prop).
|  (∀x,y∈[x / L].P(x,y)) ⟺ (∀x,y∈L.P(x,y)) & (∀y∈L.P(x,y))
| by Auto THEN All (Unfold ‘pairwise‘)
|
|\
| 1. T : Type
| 2. x : T
| 3. L : T List
| 4. P : T→T→Prop
| 5. ∀i:ℕ||[x / L]||, j:ℕi. P([x / L][j],[x / L][i])
| 6. i : ℕ||L||
| 7. j : ℕi
| ⊢  P(L[j],L[i])
|   by InstHyp [i+1;j+1] -3
|      THEN
|      RWO
|      Thm* ∀x:T, l:T List, i:ℕ||l||. [x / l][(i+1)] ∼ l[i]
|      {select_cons_tl_sq}
|      -1
|
|\
| 1. T : Type
| 2. x : T
| 3. L : T List
| 4. P : T→T→Prop
| 5. ∀i:ℕ||[x / L]||, j:ℕi. P([x / L][j],[x / L][i])
| ⊢  (∀y∈L.P(x,y))  by Auto
| |
| 1. T : Type
| 2. x : T
| 3. L : T List
| 4. P : T→T→Prop
| 5. ∀i:ℕ||[x / L]||, j:ℕi. P([x / L][j],[x / L][i])
| ⊢  (∀y∈L.P(x,y))
| | by Unfold ‘l_all‘ 0 THEN Unfold ‘l_member‘ -1 THEN ExRepD
| |
| 1. T : Type
| 2. x : T
| 3. L : T List
| 4. P : T→T→Prop
| 5. ∀i:ℕ||[x / L]||, j:ℕi. P([x / L][j],[x / L][i])
| 6. y : T
| 7. i : ℕ
| 8. i<||L||
| 9. y = L[i]
| ⊢  P(x,y)
| | by InstHyp [i+1;0] -5 THEN Reduce -1
| |    THEN
| |    RWO
| |    Thm* ∀x:T, l:T List, i:ℕ||l||. [x / l][(i+1)] ∼ l[i]
| |    {select_cons_tl_sq}
| |    -1
| |
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| 1. T : Type
| 2. x : T
| 3. L : T List
| 4. P : T→T→Prop
| 5. ∀i:ℕ||[x / L]||, j:ℕi. P([x / L][j],[x / L][i])
| 6. y : T
| 7. i : ℕ
| 8. i<||L||
| 9. y = L[i]
| 10. P(x,L[i])
| ⊢  P(x,y)  by RevHypSubst -2 -1
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. 1. T : Type
. 2. x : T
. 3. L : T List
. 4. P : T→T→Prop
. 5. ∀i:ℕ||L||, j:ℕi. P(L[j],L[i])
. 6. (∀y∈L.P(x,y))
. 7. i : ℕ||[x / L]||
. 8. j : ℕi
. ⊢  P([x / L][j],[x / L][i])
. | by CaseNat 0 ‘j’ THEN Reduce 0
. |    THEN
. |    Subst (i ∼ (i-1+1)) 0 THENL [Assert (i ∈ ℤ);Id]
. |    THEN
. |    RWO
. |    Thm* ∀x:T, l:T List, i:ℕ||l||. [x / l][(i+1)] ∼ l[i]
. |    {select_cons_tl_sq}
. |    0
. |
. |\
. | 1. T : Type
. | 2. x : T
. | 3. L : T List
. | 4. P : T→T→Prop
. | 5. ∀i:ℕ||L||, j:ℕi. P(L[j],L[i])
. | 6. (∀y∈L.P(x,y))
. | 7. i : ℕ||[x / L]||
. | 8. j : ℕi
. | 9. j = 0
. | ⊢  P(x,L[(i-1)])  by Unfold ‘l_all‘ -4 THEN BackThru -4
. | |
. | 1. T : Type
. | 2. x : T
. | 3. L : T List
. | 4. P : T→T→Prop
. | 5. ∀i:ℕ||L||, j:ℕi. P(L[j],L[i])
. | 6. ∀y:T. (y ∈ L) ⇒ P(x,y)
. | 7. i : ℕ||[x / L]||
. | 8. j : ℕi
. | 9. j = 0
. | ⊢  (L[(i-1)] ∈ L)
. |   by BackThru
. |      Thm* ∀L:T List, i:ℕ||L||. (L[i] ∈ L)    {select_member}
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. . 1. T : Type
. . 2. x : T
. . 3. L : T List
. . 4. P : T→T→Prop
. . 5. ∀i:ℕ||L||, j:ℕi. P(L[j],L[i])
. . 6. (∀y∈L.P(x,y))
. . 7. i : ℕ||[x / L]||
. . 8. j : ℕi
. . 9. ¬j = 0
. . ⊢  P([x / L][j],L[(i-1)])
. . | by Subst (j ∼ (j-1+1)) 0 THENL [Assert (j ∈ ℤ);Id]
. . |    THEN
. . |    RWO
. . |    Thm* ∀x:T, l:T List, i:ℕ||l||. [x / l][(i+1)] ∼ l[i]
. . |    {select_cons_tl_sq}
. . |    0
. . |
. . 1. T : Type
. . 2. x : T
. . 3. L : T List
. . 4. P : T→T→Prop
. . 5. ∀i:ℕ||L||, j:ℕi. P(L[j],L[i])
. . 6. (∀y∈L.P(x,y))
. . 7. i : ℕ||[x / L]||
. . 8. j : ℕi
. . 9. ¬j = 0
. . ⊢  P(L[(j-1)],L[(i-1)])  by BackThruSomeHyp
END
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