```
decidable__es-locl  THM

⊢  ∀the_es:ES, e',e:E. Dec((e <loc e'))
| by LocLessInd THEN (Decide first(j))
|
|\
| 1. the_es : ES
| 2. WellFnd{i}(E;x,y.(x <loc y))
| 3. j : E
| 4. ∀k:E. (k <loc j) ⇒ (∀e:E. Dec((e <loc k)))
| 5. first(j)
| ⊢  ∀e:E. Dec((e <loc j))
| | by Auto THEN Unfold `decidable` 0 THEN OrRight
| |
| 1. the_es : ES
| 2. WellFnd{i}(E;x,y.(x <loc y))
| 3. j : E
| 4. ∀k:E. (k <loc j) ⇒ (∀e:E. Dec((e <loc k)))
| 5. first(j)
| 6. e : E
| ⊢  ¬(e <loc j)
|   by BackThru
|      Thm* ∀the_es:ES, j,e:E. first(j) ⇒ ¬(e <loc j)
|      {es_DASH_first_DASH_implies}
|
.\
. 1. the_es : ES
. 2. WellFnd{i}(E;x,y.(x <loc y))
. 3. j : E
. 4. ∀k:E. (k <loc j) ⇒ (∀e:E. Dec((e <loc k)))
. 5. ¬first(j)
. ⊢  ∀e:E. Dec((e <loc j))
. | by Auto
. |    THEN
. |    RWO
. |    Thm* ∀the_es:ES, e,e':E.
. |         (e <loc e')
. |         ⟺
. |         ¬first(e') & e = pred(e') ∈ E ∨ (e <loc pred(e'))
. |    {es_DASH_locl_DASH_iff}
. |    0
. |    THEN
. |    BackThru
. |    Thm* Dec(P) ⇒ Dec(Q) ⇒ Dec(P & Q)    {decidable__and}
. |    THEN
. |    Try (Complete Auto)
. |    THEN
. |    BackThru Thm* Dec(P) ⇒ Dec(Q) ⇒ Dec(P ∨ Q)    {decidable__or}
. |    THEN
. |    Try (Complete Auto)
. |
. 1. the_es : ES
. 2. WellFnd{i}(E;x,y.(x <loc y))
. 3. j : E
. 4. ∀k:E. (k <loc j) ⇒ (∀e:E. Dec((e <loc k)))
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. 5. ¬first(j)
. 6. e : E
. ⊢  Dec((e <loc pred(j)))  by BackThru -3
. |
. 1. the_es : ES
. 2. WellFnd{i}(E;x,y.(x <loc y))
. 3. j : E
. 4. ∀k:E. (k <loc j) ⇒ (∀e:E. Dec((e <loc k)))
. 5. ¬first(j)
. 6. E
. ⊢  (pred(j) <loc j)
.   by BackThru
.      Thm* ∀the_es:ES, j:E. ¬first(j) ⇒ (pred(j) <loc j)
.      {es_DASH_pred_DASH_locl}
END
```