```
lpath-members-connected  THM

⊢  ∀p:IdLnk List, i:ℕ||p||, j:ℕ(i+1).
|  lpath(p) ⇒ lconnects(l_interval(p;j;i);source(p[j]);source(p[i]))
| by Auto
|
1. p : IdLnk List
2. i : ℕ||p||
3. j : ℕ(i+1)
4. lpath(p)
⊢  lconnects(l_interval(p;j;i);source(p[j]);source(p[i]))
| by Unfold ‘lconnects‘ 0 THEN Analyze 0
|
|\
| 1. p : IdLnk List
| 2. i : ℕ||p||
| 3. j : ℕ(i+1)
| 4. lpath(p)
| ⊢  lpath(l_interval(p;j;i))
|   by ParallelOp -1
|      THEN
|      Inst
|      Thm* ∀l:T List, i:ℕ||l||, j:ℕ(i+1).
|           ||l_interval(l;j;i)|| = i-j ∈ ℤ
|      {length_l_interval}
|      [IdLnk;p;i;j]
|      THEN
|      RWO
|      Thm* ∀l:T List, i:ℕ||l||, j:ℕ(i+1), x:ℕ(i-j).
|           l_interval(l;j;i)[x] = l[(j+x)]
|      {select_l_interval}
|      0
|      THEN
|      Reduce 0
|      THEN
|      InstHyp [j+i@0] -3
|      THEN
|      All ArithSimp
|
.\
. 1. p : IdLnk List
. 2. i : ℕ||p||
. 3. j : ℕ(i+1)
. 4. lpath(p)
. ⊢  (||l_interval(p;j;i)|| = 0 ⇒ source(p[j]) = source(p[i]))
. |  & (¬||l_interval(p;j;i)|| = 0
. |     ⇒
. |     source(p[j]) = source(hd(l_interval(p;j;i)))
. |     & source(p[i]) = destination(last(l_interval(p;j;i))))
. | by Auto
. |    THEN
. |    RWO
. |    Thm* ∀l:T List, i:ℕ||l||, j:ℕ(i+1).
. |         ||l_interval(l;j;i)|| = i-j ∈ ℤ
. |    {length_l_interval}
```

```
. |    -1
. |
. |\
. | 1. p : IdLnk List
. | 2. i : ℕ||p||
. | 3. j : ℕ(i+1)
. | 4. lpath(p)
. | 5. i-j = 0
. | ⊢  source(p[j]) = source(p[i])  by Subst' (i = j) 0
. |
. |\
. | 1. p : IdLnk List
. | 2. i : ℕ||p||
. | 3. j : ℕ(i+1)
. | 4. lpath(p)
. | 5. ¬i-j = 0
. | ⊢  source(p[j]) = source(hd(l_interval(p;j;i)))
. |   by RWO
. |      Thm* ∀l:T List, i:ℕ||l||, j:ℕi. hd(l_interval(l;j;i)) = l[j]
. |      {hd_l_interval}
. |      0
. |
. .\
. . 1. p : IdLnk List
. . 2. i : ℕ||p||
. . 3. j : ℕ(i+1)
. . 4. lpath(p)
. . 5. ¬i-j = 0
. . ⊢  source(p[i]) = destination(last(l_interval(p;j;i)))
. . | by RWO
. . |    Thm* ∀l:T List, i:ℕ||l||, j:ℕi.
. . |         last(l_interval(l;j;i)) = l[(i-1)]
. . |    {last_l_interval}
. . |    0
. . |
. . 1. p : IdLnk List
. . 2. i : ℕ||p||
. . 3. j : ℕ(i+1)
. . 4. lpath(p)
. . 5. ¬i-j = 0
. . ⊢  source(p[i]) = destination(p[(i-1)])
. . | by Unfold `lpath` -2 THEN InstHyp [i-1] -2
. . |
. . 1. p : IdLnk List
. . 2. i : ℕ||p||
. . 3. j : ℕ(i+1)
. . 4. ∀i:ℕ(||p||-1).
. .    destination(p[i]) = source(p[(i+1)])
. .    & ¬p[(i+1)] = lnk-inv(p[i])
. . 5. ¬i-j = 0
. . 6. destination(p[(i-1)]) = source(p[(i-1+1)])
. . 7. ¬p[(i-1+1)] = lnk-inv(p[(i-1)])
. . ⊢  source(p[i]) = destination(p[(i-1)])
. .   by Symmetry THEN NthHypEq -2 THEN Analyze
END
```