```
lpath_cons  THM

⊢  ∀l:IdLnk, p:IdLnk List.
|  lpath([l / p])
|  ⟺
|  lpath(p)
|  & (¬||p|| = 0
|     ⇒
|     destination(l) = source(hd(p)) & ¬hd(p) = lnk-inv(l))  by Auto
|
|\
| 1. l : IdLnk
| 2. p : IdLnk List
| 3. lpath([l / p])
| ⊢  lpath(p)
|   by ParallelOp -1 THEN Analyze 0 THEN InstHyp [i+1] -2
|      THEN
|      RWO
|      Thm* ∀a:T, as:T List, i:ℤ.
|           0<i ⇒ i≤||as|| ⇒ [a / as][i] = as[(i-1)]
|      {select_cons_tl}
|      -1
|      THEN
|      Subst ((i+1-1) ∼ i) -1
|      THENL
|      [Auto;Id]
|      THEN
|      Subst ((i+1+1-1) ∼ (i+1)) -1
|
|\
| 1. l : IdLnk
| 2. p : IdLnk List
| 3. lpath([l / p])
| 4. ¬||p|| = 0
| ⊢  destination(l) = source(hd(p))
|   by Unfold `lpath` -2 THEN InstHyp [0] -2 THEN Reduce -1
|      THEN
|      NthHypEq -2
|      THEN
|      Analyze
|      THEN
|      Analyze
|      THEN
|      DVar `p'
|      THEN
|      All Reduce
|
|\
| 1. l : IdLnk
| 2. p : IdLnk List
| 3. lpath([l / p])
| 4. ¬||p|| = 0
| ⊢  ¬hd(p) = lnk-inv(l)
|   by Unfold `lpath` -2 THEN InstHyp [0] -2 THEN Reduce -1
|      THEN
```

```
|      NthHypEq -1
|      THEN
|      Analyze
|      THEN
|      Analyze
|      THEN
|      DVar ‘p’
|      THEN
|      All Reduce
|
.\
. 1. l : IdLnk
. 2. p : IdLnk List
. 3. lpath(p)
. 4. ¬||p|| = 0
. |  ⇒
. |  destination(l) = source(hd(p)) & ¬hd(p) = lnk-inv(l)
. ⊢  lpath([l / p])  by DVar ‘p’
. |
. |\
. | 1. l : IdLnk
. | 2. lpath(nil)
. | 3. ¬||nil|| = 0
. |    ⇒
. |    destination(l) = source(hd(nil)) & ¬hd(nil) = lnk-inv(l)
. | ⊢  lpath([l])  by Unfold ‘lpath‘ 0 THEN Reduce 0
. |
. .\
. . 1. l : IdLnk
. . 2. u : IdLnk
. . 3. v : IdLnk List
. . 4. lpath([u / v])
. . 5. ¬||[u / v]|| = 0
. . |  ⇒
. . |  destination(l) = source(hd([u / v]))
. . |  & ¬hd([u / v]) = lnk-inv(l)
. . ⊢  lpath([l; u / v])  by Analyze -1 THEN Reduce -1
. . |
. . 1. l : IdLnk
. . 2. u : IdLnk
. . 3. v : IdLnk List
. . 4. lpath([u / v])
. . 5. destination(l) = source(u)
. . 6. ¬u = lnk-inv(l)
. . ⊢  lpath([l; u / v])
. . | by ParallelOp -3 THEN Analyze 0 THEN CaseNat 0 ‘i’ THEN Reduce 0
. . |    THEN
. . |    RWO
. . |    Thm* ∀a:T, as:T List, i:ℤ.
. . |         0<i ⇒ i≤||as|| ⇒ [a / as][i] = as[(i-1)]
. . |    {select_cons_tl}
. . |    0
. . |    THEN
. . |    InstHyp [i-1] 4
. . |
```

```
. . |\
. . | 1. l : IdLnk
. . | 2. u : IdLnk
. . | 3. v : IdLnk List
. . | 4. ∀i:ℕ(||[u / v]||-1).
. . |    destination([u / v][i]) = source([u / v][(i+1)])
. . |    & ¬[u / v][(i+1)] = lnk-inv([u / v][i])
. . | 5. destination(l) = source(u)
. . | 6. ¬u = lnk-inv(l)
. . | 7. i : ℕ(||[l; u / v]||-1)
. . | 8. ¬i = 0
. . | 9. destination([u / v][(i-1)]) = source([u / v][(i-1+1)])
. . | 10. ¬[u / v][(i-1+1)] = lnk-inv([u / v][(i-1)])
. . | ⊢  destination([u / v][(i-1)]) = source([u / v][(i+1-1)])
. . |   by NthHypEq -2 THEN Analyze
. . |
. . .\
. . . 1. l : IdLnk
. . . 2. u : IdLnk
. . . 3. v : IdLnk List
. . . 4. ∀i:ℕ(||[u / v]||-1).
. . .    destination([u / v][i]) = source([u / v][(i+1)])
. . .    & ¬[u / v][(i+1)] = lnk-inv([u / v][i])
. . . 5. destination(l) = source(u)
. . . 6. ¬u = lnk-inv(l)
. . . 7. i : ℕ(||[l; u / v]||-1)
. . . 8. ¬i = 0
. . . 9. destination([u / v][(i-1)]) = source([u / v][(i-1+1)])
. . . 10. ¬[u / v][(i-1+1)] = lnk-inv([u / v][(i-1)])
. . . ⊢  ¬[u / v][(i+1-1)] = lnk-inv([u / v][(i-1)])
. . .   by NthHypEq -1 THEN Analyze
END
```