```
assert-w-first  THM

⊢  ∀the_w:World, e:E.
|  first(e) ⟺ (∀t':ℕ. t'<time(e) ⇒ isnull(a(loc(e);t')))
| by Assert
|    (∀the_w:World, i:Id, t:ℕ.
|     first(<i,t>) ⟺ (∀t':ℕ. t'<t ⇒ isnull(a(i;t'))))
|
|\
| .....assertion .....
| ⊢  ∀the_w:World, i:Id, t:ℕ.
| |  first(<i,t>) ⟺ (∀t':ℕ. t'<t ⇒ isnull(a(i;t')))
| | by InductionOnNat THEN RecUnfold `w-first` 0
| |    THEN
| |    Unfolds [`w-time`;`w-loc`] 0
| |    THEN
| |    Reduce 0
| |
| |\
| | 1. the_w : World
| | 2. i : Id
| | ⊢  True ⟺ (∀t':ℕ. t'<0 ⇒ isnull(a(i;t')))  by Auto
| |
| .\
| . 1. the_w : World
| . 2. i : Id
| . 3. t : ℤ
| . 4. 0<t
| . 5. first(<i,t-1>) ⟺ (∀t':ℕ. t'<t-1 ⇒ isnull(a(i;t')))
| . ⊢  if t=₂0→ true₂
| . |   ; isnull(a(i;t-1))→ first(<i,t-1>)
| . |  else false₂ fi
| . |  ⟺
| . |  (∀t':ℕ. t'<t ⇒ isnull(a(i;t')))
| . | by SplitOnConclITE THENL [Auto;SplitOnConclITE]
| . |
| . |\
| . | .....truecase .....
| . | 1. the_w : World
| . | 2. i : Id
| . | 3. t : ℤ
| . | 4. 0<t
| . | 5. first(<i,t-1>) ⟺ (∀t':ℕ. t'<t-1 ⇒ isnull(a(i;t')))
| . | 6. ¬t=₂0
| . | 7. isnull(a(i;t-1))
| . | ⊢  first(<i,t-1>) ⟺ (∀t':ℕ. t'<t ⇒ isnull(a(i;t')))
| . | | by AssertBY (first(<i,t-1>) ∈ 𝔹)
| . | |    (BackThru
| . | |     Thm* ∀the_w:World, i:Id, t:ℕ. first(<i,t>) ∈ 𝔹
| . | |     {w_DASH_first_DASH_aux})
| . | |
| . | 1. the_w : World
| . | 2. i : Id
| . | 3. t : ℤ
| . | 4. 0<t
```

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| . | 5. first(<i,t-1>) ⟺ (∀t':ℕ. t'<t-1 ⇒ isnull(a(i;t')))
| . | 6. ¬t=_2 0
| . | 7. isnull(a(i;t-1))
| . | 8. first(<i,t-1>) ∈ 𝔹
| . | ⊢  first(<i,t-1>) ⟺ (∀t':ℕ. t'<t ⇒ isnull(a(i;t')))
| . | | by ParallelOp -4
| . | |
| . | |\
| . | | 1. the_w : World
| . | | 2. i : Id
| . | | 3. t : ℤ
| . | | 4. 0<t
| . | | 5. ¬t=_2 0
| . | | 6. isnull(a(i;t-1))
| . | | 7. first(<i,t-1>) ∈ 𝔹
| . | | 8. first(<i,t-1>)
| . | | 9. ∀t':ℕ. t'<t-1 ⇒ isnull(a(i;t'))
| . | | ⊢  ∀t':ℕ. t'<t ⇒ isnull(a(i;t'))
| . | | | by Auto THEN Decide (t'<t-1)
| . | | |
| . | | |\
| . | | | 1. the_w : World
| . | | | 2. i : Id
| . | | | 3. t : ℤ
| . | | | 4. 0<t
| . | | | 5. ¬t=_2 0
| . | | | 6. isnull(a(i;t-1))
| . | | | 7. first(<i,t-1>) ∈ 𝔹
| . | | | 8. first(<i,t-1>)
| . | | | 9. ∀t':ℕ. t'<t-1 ⇒ isnull(a(i;t'))
| . | | | 10. t' : ℕ
| . | | | 11. t'<t
| . | | | 12. t'<t-1
| . | | | ⊢  isnull(a(i;t'))  by BackThru -4
| . | | |
| . | | .\
| . | | . 1. the_w : World
| . | | . 2. i : Id
| . | | . 3. t : ℤ
| . | | . 4. 0<t
| . | | . 5. ¬t=_2 0
| . | | . 6. isnull(a(i;t-1))
| . | | . 7. first(<i,t-1>) ∈ 𝔹
| . | | . 8. first(<i,t-1>)
| . | | . 9. ∀t':ℕ. t'<t-1 ⇒ isnull(a(i;t'))
| . | | . 10. t' : ℕ
| . | | . 11. t'<t

| . | | . 12. ¬t'<t-1
| . | | . ⊢  isnull(a(i;t'))  by Subst' (t' = t-1) 0
| . | |
| . | .\
| . | . .....antecedent .....
| . | . 1. the_w : World
| . | . 2. i : Id
| . | . 3. t : ℤ
```

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| . | . 4. 0<t
| . | . 5. ¬t=_2 0
| . | . 6. isnull(a(i;t-1))
| . | . 7. first(<i,t-1>) ∈ 𝔹
| . | . 8. ∀t':ℕ. t'<t ⇒ isnull(a(i;t'))
| . | . ⊢  ∀t':ℕ. t'<t-1 ⇒ isnull(a(i;t'))
| . | .   by RepeatFor 2 (ParallelOp -1)
| . |
| . .\
| . . .....falsecase .....
| . . 1. the_w : World
| . . 2. i : Id
| . . 3. t : ℤ
| . . 4. 0<t
| . . 5. first(<i,t-1>) ⟺ (∀t':ℕ. t'<t-1 ⇒ isnull(a(i;t')))
| . . 6. ¬t=_2 0
| . . 7. ¬isnull(a(i;t-1))
| . . ⊢  false_2 ⟺ (∀t':ℕ. t'<t ⇒ isnull(a(i;t')))
| . .   by Reduce 0 THEN InstHyp [t-1] -1
|
.\
. 1. ∀the_w:World, i:Id, t:ℕ.
. |  first(<i,t>) ⟺ (∀t':ℕ. t'<t ⇒ isnull(a(i;t')))
. ⊢  ∀the_w:World, e:E.
. |  first(e) ⟺ (∀t':ℕ. t'<time(e) ⇒ isnull(a(loc(e);t')))
. | by UnivCD
. |
. 1. ∀the_w:World, i:Id, t:ℕ.
.    first(<i,t>) ⟺ (∀t':ℕ. t'<t ⇒ isnull(a(i;t')))
. 2. the_w : World
. 3. e : E
. ⊢  first(e) ⟺ (∀t':ℕ. t'<time(e) ⇒ isnull(a(loc(e);t')))
.   by Analyze -1 THEN Analyze -2 THEN InstHyp [the_w;e1;e2] 1
.      THEN
.      Try (Unfolds [`w-time`;`w-loc`] 0 THEN Reduce 0)
.      THEN
.      Trivial
END
```