```
better-w-match-exists  THM

⊢  ∀the_w:World, e:E.
|  FairFifo
|  ⇒
|  isrcv(kind(e))
|  ⇒
|  (∃t:ℕtime(e).
|   match(lnk(kind(e));t;time(e))
|   & onlnk(lnk(kind(e));m(source(lnk(kind(e)));t))[(||rcvs(...;...)||
|     -||snds(lnk(kind(e));t)||)]
|     =
|     msg(a(loc(e);time(e))))
| by Auto THEN Unfold ‘fair-fifo‘ 3
|    THEN
|    Assert isrcv(lnk(kind(e));a(loc(e);time(e)))
|
|\
| .....assertion .....
| 1. the_w : World
| 2. e : E
| 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
| 4. ∀i:Id, t:ℕ.
| |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
| 5. ∀i:Id, t:ℕ, l:IdLnk.
| |  isrcv(l;a(i;t))
| |  ⇒
| |  destination(l) = i
| |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
| 6. ∀l:IdLnk, t:ℕ.
| |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
| 7. isrcv(kind(e))
| ⊢  isrcv(lnk(kind(e));a(loc(e);time(e)))
| | by RepeatFor 4 (Thin 3) THEN Unfold ‘w-isrcvl‘ 0 THEN Analyze 2
| |    THEN
| |    Analyze 2
| |    THEN
| |    All (Unfolds [‘w-loc‘;‘w-time‘;‘w-ekind‘])
| |    THEN
| |    All Reduce
| |    THEN
| |    All (Unfolds [‘w-act‘])
| |    THEN
| |    All Reduce
| |
| 1. the_w : World
| 2. e1 : Id
| 3. e2 : ℕ
| 4. ¬isnull(a(e1;e2))  [not for witness]
| 5. isrcv(kind(a(e1;e2)))
| ⊢  ¬₂isnull(a(e1;e2))∧₂isrcv(kind(a(e1;e2)))∧₂
| |  lnk(kind(a(e1;e2))) = lnk(kind(a(e1;e2)))
| | by Unhide THEN AutoBoolCase isnull(a(e1;e2))
| |    THEN
| |    AutoBoolCase isrcv(kind(a(e1;e2)))
```

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| |
| 1. the_w : World
| 2. e1 : Id
| 3. e2 : ℕ
| 4. ¬isnull(a(e1;e2))
| 5. isrcv(kind(a(e1;e2)))
| 6. ¬isnull(a(e1;e2))
| 7. isrcv(kind(a(e1;e2)))
| ⊢  lnk(kind(a(e1;e2))) = lnk(kind(a(e1;e2)))
|   by RW assert_pushdownC 0
|
.\
. 1. the_w : World
. 2. e : E
. 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. 4. ∀i:Id, t:ℕ.
. |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. 5. ∀i:Id, t:ℕ, l:IdLnk.
. |  isrcv(l;a(i;t))
. |  ⇒
. |  destination(l) = i
. |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. 6. ∀l:IdLnk, t:ℕ.
. |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. 7. isrcv(kind(e))
. 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. ⊢  ∃t:ℕtime(e).
. |  match(lnk(kind(e));t;time(e))
. |  & onlnk(lnk(kind(e));m(source(lnk(kind(e)));t))[(||...||
. |    -||snds(lnk(kind(e));t)||)]
. |    =
. |    msg(a(loc(e);time(e)))
. | by InstHyp [loc(e);time(e);lnk(kind(e))] 5
. |
. ...
. |
. |\
. | .....assertion .....
. | 1. the_w : World
. | 2. e : E
. | 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. | 4. ∀i:Id, t:ℕ.
. | |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. | 5. ∀i:Id, t:ℕ, l:IdLnk.
. | |  isrcv(l;a(i;t))
. | |  ⇒
. | |  destination(l) = i
. | |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. | 6. ∀l:IdLnk, t:ℕ.
. | |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. | 7. isrcv(kind(e))
. | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. | 9. destination(lnk(kind(e))) = loc(e)
. | 10. ||queue(lnk(kind(e));time(e))||≥1
. | |   & hd(queue(lnk(kind(e));time(e))) = msg(a(loc(e);time(e)))
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. | ⊢  ||rcvs(lnk(kind(e));time(e))||<||snds(lnk(kind(e));time(e))||
. | | by ExRepD THEN Thin -1 THEN MoveToConcl -1
. | |    THEN
. | |    Unfold `w-queue` 0
. | |    THEN
. | |    GenConcl (||rcvs(lnk(kind(e));time(e))|| = r)
. | |    THEN
. | |    GenConcl (snds(lnk(kind(e));time(e)) = L)
. | |
. | 1. the_w : World
. | 2. e : E
. | 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. | 4. ∀i:Id, t:ℕ.
. |    isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. | 5. ∀i:Id, t:ℕ, l:IdLnk.
. |    isrcv(l;a(i;t))
. |    ⇒
. |    destination(l) = i
. |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. | 6. ∀l:IdLnk, t:ℕ.
. |    ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. | 7. isrcv(kind(e))
. | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. | 9. destination(lnk(kind(e))) = loc(e)
. | 10. r : ℕ
. | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. | 12. L : Top List
. | 13. snds(lnk(kind(e));time(e)) = L
. | ⊢  ||nth_tl(r;L)||≥1 ⇒ r<||L||
. |   by (RWO
. |       Thm* ∀r:ℕ, L:Top List.
. |            ||nth_tl(r;L)|| = if r<₂||L||→ ||L||-r else 0 fi
. |       {general_length_nth_tl}
. |       0)
. |      THEN
. |      SplitOnConclITE
. |
. .\
. . 1. the_w : World
. . 2. e : E
. . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . 4. ∀i:Id, t:ℕ.
. . |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . |  isrcv(l;a(i;t))
. . |  ⇒
. . |  destination(l) = i
. . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . 6. ∀l:IdLnk, t:ℕ.
. . |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . 7. isrcv(kind(e))
. . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . 9. destination(lnk(kind(e))) = loc(e)
. . 10. ||queue(lnk(kind(e));time(e))||≥1
. . |   & hd(queue(lnk(kind(e));time(e))) = msg(a(loc(e);time(e)))
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. . 11. ||rcvs(lnk(kind(e));time(e))||<||snds(lnk(kind(e));time(e))||
. . ⊢  ∃t:ℕtime(e).
. . |  match(lnk(kind(e));t;time(e))
. . |  & onlnk(lnk(kind(e));m(source(lnk(kind(e)));t))[(||...||
. . |    -||snds(lnk(kind(e));t)||)]
. . |    =
. . |    msg(a(loc(e);time(e)))
. . | by Unfold `w-queue` -2 THEN ExRepD THEN Thin -3
. . |    THEN
. . |    RWO
. . |    Thm* ∀m:ℕ, L:T List.
. . |         m<||L|| ⇒ (nth_tl(m;L) ∼ [L[m] / nth_tl(1+m;L)])
. . |    {nth_tl_decomp}
. . |    -2
. . |    THEN
. . |    Reduce -2
. . |
. . 1. the_w : World
. . 2. e : E
. . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . 4. ∀i:Id, t:ℕ.
. . |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . |  isrcv(l;a(i;t))
. . |  ⇒
. . |  destination(l) = i
. . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . 6. ∀l:IdLnk, t:ℕ.
. . |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . 7. isrcv(kind(e))
. . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . 9. destination(lnk(kind(e))) = loc(e)
. . 10. snds(lnk(kind(e));time(e))[||rcvs(lnk(kind(e));time(e))||]
. . |   =
. . |   msg(a(loc(e);time(e)))
. . 11. ||rcvs(lnk(kind(e));time(e))||<||snds(lnk(kind(e));time(e))||
. . ⊢  ∃t:ℕtime(e).

. . |  match(lnk(kind(e));t;time(e))
. . |  & onlnk(lnk(kind(e));m(source(lnk(kind(e)));t))[(||...||
. . |    -||snds(lnk(kind(e));t)||)]
. . |    =
. . |    msg(a(loc(e);time(e)))
. . | by MoveToConcl -2 THEN MoveToConcl -1
. . |    THEN
. . |    GenConcl (||rcvs(lnk(kind(e));time(e))|| = r)
. . |    THEN
. . |    GenConcl (msg(a(loc(e);time(e))) = ms)
. . |    THENA
. . |    Using ['l',lnk(kind(e))] Auto
. . |
. . 1. the_w : World
. . 2. e : E
. . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . 4. ∀i:Id, t:ℕ.
. . |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
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. . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . |  isrcv(l;a(i;t))
. . |  ⇒
. . |  destination(l) = i
. . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . 6. ∀l:IdLnk, t:ℕ.
. . |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . 7. isrcv(kind(e))
. . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . 9. destination(lnk(kind(e))) = loc(e)
. . 10. r : ℕ
. . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . 12. ms : Msg
. . 13. msg(a(loc(e);time(e))) = ms
. . ⊢  r<||snds(lnk(kind(e));time(e))||
. . |  ⇒
. . |  snds(lnk(kind(e));time(e))[r] = ms
. . |  ⇒
. . |  (∃t:ℕtime(e).
. . |   match(lnk(kind(e));t;time(e))
. . |   & onlnk(lnk(kind(e));m(source(lnk(kind(e)));t))[(r
. . |     -||snds(lnk(kind(e));t)||)]
. . |     =
. . |     ms)
. . | by Unfold `w-match` 0 THEN HypSubst' -3 0
. . |    THEN
. . |    GenConcl (lnk(kind(e)) = l)
. . |    THEN
. . |    GenConcl (time(e) = t')
. . |
. . 1. the_w : World
. . 2. e : E
. . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . 4. ∀i:Id, t:ℕ.
. . |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . |  isrcv(l;a(i;t))
. . |  ⇒
. . |  destination(l) = i
. . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . 6. ∀l:IdLnk, t:ℕ.
. . |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . 7. isrcv(kind(e))
. . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . 9. destination(lnk(kind(e))) = loc(e)
. . 10. r : ℕ
. . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . 12. ms : Msg
. . 13. msg(a(loc(e);time(e))) = ms
. . 14. l : IdLnk
. . 15. lnk(kind(e)) = l
. . 16. t' : ℕ
. . 17. time(e) = t'
. . ⊢  r<||snds(l;t')||
. . |  ⇒
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. . |  snds(l;t')[r] = ms
. . |  ⇒
. . |  (∃t:ℕt'.
. . |   ((||snds(l;t)||≤₂r)∧₂
. . |    (r<₂||snds(l;t)||+||onlnk(l;m(source(l);t))||))
. . |   & onlnk(l;m(source(l);t))[(r-||snds(l;t)||)] = ms)
. . | by Auto THEN Unfold ‘and‘ 0 THEN Fold ‘cand‘ 0
. . |
. . 1. the_w : World
. . 2. e : E
. . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . 4. ∀i:Id, t:ℕ.
. . |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . |  isrcv(l;a(i;t))
. . |  ⇒
. . |  destination(l) = i
. . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . 6. ∀l:IdLnk, t:ℕ.
. . |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . 7. isrcv(kind(e))
. . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . 9. destination(lnk(kind(e))) = loc(e)
. . 10. r : ℕ
. . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . 12. ms : Msg
. . 13. msg(a(loc(e);time(e))) = ms

. . 14. l : IdLnk
. . 15. lnk(kind(e)) = l
. . 16. t' : ℕ
. . 17. time(e) = t'
. . 18. r<||snds(l;t')||
. . 19. snds(l;t')[r] = ms
. . ⊢  ∃t:ℕt'.
. . |  ((||snds(l;t)||≤₂r)∧₂
. . |   (r<₂||snds(l;t)||+||onlnk(l;m(source(l);t))||))
. . |  & onlnk(l;m(source(l);t))[(r-||snds(l;t)||)] = ms
. . | by Unfold ‘w-snds‘ -1
. . |    THEN
. . |    Inst Thm: select_concat [Msg;map(λt1.m(l;t1);upto(t'));r]
. . |
. . |\
. . | 1. the_w : World
. . | 2. e : E
. . | 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . | 4. ∀i:Id, t:ℕ.
. . |    isnull(a(i;t))
. . |    ⇒
. . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . |    isrcv(l;a(i;t))
. . |    ⇒
. . |    destination(l) = i
. . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . | 6. ∀l:IdLnk, t:ℕ.
```

```
. . |    ∃t':ℕ.
. . |    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . | 7. isrcv(kind(e))
. . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . | 9. destination(lnk(kind(e))) = loc(e)
. . | 10. r : ℕ
. . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . | 12. ms : Msg
. . | 13. msg(a(loc(e);time(e))) = ms
. . | 14. l : IdLnk
. . | 15. lnk(kind(e)) = l
. . | 16. t' : ℕ
. . | 17. time(e) = t'
. . | 18. r<||snds(l;t')||
. . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . | ⊢  r<||concat(map(λt1.m(l;t1);upto(t')))||
. . |   by Fold `w-snds` 0 THEN Trivial
. . |
. . .\
. . . 1. the_w : World
. . . 2. e : E
. . . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . 4. ∀i:Id, t:ℕ.
. . . |  isnull(a(i;t))
. . . |  ⇒
. . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . |  isrcv(l;a(i;t))
. . . |  ⇒
. . . |  destination(l) = i
. . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . 6. ∀l:IdLnk, t:ℕ.
. . . |  ∃t':ℕ.
. . . |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . 7. isrcv(kind(e))
. . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . 9. destination(lnk(kind(e))) = loc(e)
. . . 10. r : ℕ
. . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . 12. ms : Msg
. . . 13. msg(a(loc(e);time(e))) = ms
. . . 14. l : IdLnk
. . . 15. lnk(kind(e)) = l
. . . 16. t' : ℕ
. . . 17. time(e) = t'
. . . 18. r<||snds(l;t')||
. . . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . 20. ∃m:ℕ||map(λt1.m(l;t1);upto(t'))||.
. . . |   ||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||≤r
. . . |   & r-||concat(firstn(m;map(λt1.
. . . |   m(l;t1);upto(t'))))||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . |   & concat(map(λt1.m(l;t1);upto(t')))[r]
. . . |     =
. . . |     map(λt1.m(l;t1);upto(t'))[m][(r-||concat(firstn(m;map(λt1.
. . . |     m(l;t1);upto(t'))))||)]
```

```
. . . ⊢  ∃t:ℕt'.
. . . |  ((||snds(l;t)||≤₂r)∧₂
. . . |   (r<₂||snds(l;t)||+||onlnk(l;m(source(l);t))||))
. . . |  & onlnk(l;m(source(l);t))[(r-||snds(l;t)||)] = ms
. . . | by Analyze -1 THEN Assert (m<t')
. . . |
. . . |\
. . . | .....assertion .....
. . . | 1. the_w : World
. . . | 2. e : E
. . . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . | |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . | 4. ∀i:Id, t:ℕ.
. . . | |  isnull(a(i;t))
. . . | |  ⇒
. . . | |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . | |  isrcv(l;a(i;t))
. . . | |  ⇒
. . . | |  destination(l) = i
. . . | |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . | 6. ∀l:IdLnk, t:ℕ.
. . . | |  ∃t':ℕ.
. . . | |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . | 7. isrcv(kind(e))
. . . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . | 9. destination(lnk(kind(e))) = loc(e)
. . . | 10. r : ℕ
. . . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . | 12. ms : Msg
. . . | 13. msg(a(loc(e);time(e))) = ms
. . . | 14. l : IdLnk
. . . | 15. lnk(kind(e)) = l
. . . | 16. t' : ℕ
. . . | 17. time(e) = t'
. . . | 18. r<||snds(l;t')||
. . . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . | 21. ||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||≤r
. . . | |   & r-||concat(firstn(m;map(λt1.
. . . | |   m(l;t1);upto(t'))))||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . | |   & concat(map(λt1.m(l;t1);upto(t')))[r]
. . . | |     =
. . . | |     map(λt1.m(l;t1);upto(t'))[m][(r
. . . | |     -||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||)]
. . . | ⊢  m<t'  by Subst' (||map(λt1.m(l;t1);upto(t'))|| = t') -2
. . . | |
. . . | |\
. . . | | 1. the_w : World
. . . | | 2. e : E
. . . | | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . | |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . | | 4. ∀i:Id, t:ℕ.
. . . | |    isnull(a(i;t))
. . . | |    ⇒
```

```
. . . | |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . | | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . | |    isrcv(l;a(i;t))
. . . | |    ⇒
. . . | |    destination(l) = i
. . . | |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . | | 6. ∀l:IdLnk, t:ℕ.
. . . | |    ∃t':ℕ.
. . . | |    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . | | 7. isrcv(kind(e))
. . . | | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . | | 9. destination(lnk(kind(e))) = loc(e)
. . . | | 10. r : ℕ
. . . | | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . | | 12. ms : Msg
. . . | | 13. msg(a(loc(e);time(e))) = ms
. . . | | 14. l : IdLnk
. . . | | 15. lnk(kind(e)) = l
. . . | | 16. t' : ℕ
. . . | | 17. time(e) = t'
. . . | | 18. r<||snds(l;t')||
. . . | | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . | | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . | | 21. ||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||≤r
. . . | |     & r-||concat(firstn(m;map(λt1.
. . . | |     m(l;t1);upto(t'))))||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . | |     & concat(map(λt1.m(l;t1);upto(t')))[r]
. . . | |       =
. . . | |       map(λt1.m(l;t1);upto(t'))[m][(r
. . . | |       -||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||)]
. . . | | ⊢  ||map(λt1.m(l;t1);upto(t'))|| = t'
. . . | |   by RWO
. . . | |      Thm* ∀f:(A→B), as:A List. ||map(f;as)|| = ||as|| ∈ ℤ
. . . | |      {map_length}
. . . | |      0
. . . | |      THEN
. . . | |      RWO Thm* ∀n:ℕ. ||upto(n)|| ∼ n    {length_upto} 0
. . . | |
. . . | .\
. . . | . 1. the_w : World
. . . | . 2. e : E
. . . | . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . | .    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . | . 4. ∀i:Id, t:ℕ.
. . . | .    isnull(a(i;t))
. . . | .    ⇒
. . . | .    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . | . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . | .    isrcv(l;a(i;t))
. . . | .    ⇒
. . . | .    destination(l) = i
. . . | .    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . | . 6. ∀l:IdLnk, t:ℕ.
. . . | .    ∃t':ℕ.
. . . | .    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
```

```
. . . | . 7. isrcv(kind(e))
. . . | . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . | . 9. destination(lnk(kind(e))) = loc(e)
. . . | . 10. r : ℕ
. . . | . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . | . 12. ms : Msg
. . . | . 13. msg(a(loc(e);time(e))) = ms
. . . | . 14. l : IdLnk
. . . | . 15. lnk(kind(e)) = l
. . . | . 16. t' : ℕ
. . . | . 17. time(e) = t'
. . . | . 18. r<||snds(l;t')||
. . . | . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . | . 20. m : ℕt'
. . . | . 21. ||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||≤r
. . . | .     & r-||concat(firstn(m;map(λt1.
. . . | .     m(l;t1);upto(t'))))||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . | .     & concat(map(λt1.m(l;t1);upto(t')))[r]
. . . | .       =
. . . | .       map(λt1.m(l;t1);upto(t'))[m][(r
. . . | .       -||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||)]
. . . | . ⊢  m<t'  by Auto
. . . |
. . . .\
. . . . 1. the_w : World
. . . . 2. e : E
. . . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . 4. ∀i:Id, t:ℕ.
. . . . |  isnull(a(i;t))
. . . . |  ⇒
. . . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |  isrcv(l;a(i;t))
. . . . |  ⇒
. . . . |  destination(l) = i
. . . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . 6. ∀l:IdLnk, t:ℕ.
. . . . |  ∃t':ℕ.
. . . . |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . 7. isrcv(kind(e))
. . . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . 9. destination(lnk(kind(e))) = loc(e)
. . . . 10. r : ℕ
. . . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . 12. ms : Msg
. . . . 13. msg(a(loc(e);time(e))) = ms
. . . . 14. l : IdLnk
. . . . 15. lnk(kind(e)) = l
. . . . 16. t' : ℕ
. . . . 17. time(e) = t'
. . . . 18. r<||snds(l;t')||
. . . . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . 21. ||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||≤r
```

```
. . . . |   & r-||concat(firstn(m;map(λt1.
. . . . |   m(l;t1);upto(t'))))||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . |   & concat(map(λt1.m(l;t1);upto(t')))[r]
. . . . |     =
. . . . |     map(λt1.m(l;t1);upto(t'))[m][(r
. . . . |     -||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||)]
. . . . 22. m<t'
. . . . ⊢  ∃t:ℕt'.
. . . . |  ((||snds(l;t)||≤₂r)∧₂
. . . . |   (r<₂||snds(l;t)||+||onlnk(l;m(source(l);t))||))
. . . . |  & onlnk(l;m(source(l);t))[(r-||snds(l;t)||)] = ms
. . . . | by RWO
. . . . |    Thm* ∀f:(Top→Top), n:ℕ, l:Top List.
. . . . |         firstn(n;map(f;l)) ∼ map(f;firstn(n;l))
. . . . |    {firstn_map}
. . . . |    -2
. . . . |    THEN
. . . . |    RWO
. . . . |    Thm* ∀n,m:ℕ.
. . . . |         firstn(n;upto(m)) ∼ if n≤₂m→ upto(n) else upto(m) fi
. . . . |    {firstn_upto}
. . . . |    -2
. . . . |    THEN
. . . . |    Subst ((m≤₂t') ∼ true₂) -2
. . . . |    THEN
. . . . |    Try (Complete (AutoBoolCase (m≤₂t')))
. . . . |    THEN
. . . . |    Reduce -2
. . . . |    THEN
. . . . |    Fold `w-snds` -2
. . . . |    THEN
. . . . |    ExRepD
. . . . |
. . . . 1. the_w : World
. . . . 2. e : E
. . . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . 4. ∀i:Id, t:ℕ.
. . . . |  isnull(a(i;t))
. . . . |  ⇒
. . . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |  isrcv(l;a(i;t))
. . . . |  ⇒
. . . . |  destination(l) = i
. . . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . 6. ∀l:IdLnk, t:ℕ.
. . . . |  ∃t':ℕ.
. . . . |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . 7. isrcv(kind(e))
. . . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . 9. destination(lnk(kind(e))) = loc(e)
. . . . 10. r : ℕ
. . . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . 12. ms : Msg
```

```
. . . . 13. msg(a(loc(e);time(e))) = ms
. . . . 14. l : IdLnk
. . . . 15. lnk(kind(e)) = l
. . . . 16. t' : ℕ
. . . . 17. time(e) = t'
. . . . 18. r<||snds(l;t')||
. . . . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . 21. ||snds(l;m)||≤r
. . . . 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . 23. snds(l;t')[r]
. . . . |   =
. . . . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . 24. m<t'
. . . . ⊢  ∃t:ℕt'.
. . . . |  ((||snds(l;t)||≤₂r)∧₂
. . . . |   (r<₂||snds(l;t)||+||onlnk(l;m(source(l);t))||))
. . . . |  & onlnk(l;m(source(l);t))[(r-||snds(l;t)||)] = ms
. . . . | by InstConcl [m]
. . . . |
. . . . |\
. . . . | 1. the_w : World
. . . . | 2. e : E
. . . . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | 4. ∀i:Id, t:ℕ.
. . . . | |  isnull(a(i;t))
. . . . | |  ⇒
. . . . | |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |  isrcv(l;a(i;t))
. . . . | |  ⇒
. . . . | |  destination(l) = i
. . . . | |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | 6. ∀l:IdLnk, t:ℕ.
. . . . | |  ∃t':ℕ.
. . . . | |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | 7. isrcv(kind(e))
. . . . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | 9. destination(lnk(kind(e))) = loc(e)
. . . . | 10. r : ℕ
. . . . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | 12. ms : Msg
. . . . | 13. msg(a(loc(e);time(e))) = ms
. . . . | 14. l : IdLnk
. . . . | 15. lnk(kind(e)) = l
. . . . | 16. t' : ℕ
. . . . | 17. time(e) = t'
. . . . | 18. r<||snds(l;t')||
. . . . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | 21. ||snds(l;m)||≤r
. . . . | 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | 23. snds(l;t')[r]
. . . . | |   =
```

```
. . . . | |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | 24. m<t'
. . . . | ⊢  (||snds(l;m)||≤₂r)∧₂
. . . . | |  (r<₂||snds(l;m)||+||onlnk(l;m(source(l);m))||)
. . . . | | by RW0
. . . . | |    Thm* ∀f:(A→B), as:A List, n:ℕ||as||.
. . . . | |         map(f;as)[n] = f(as[n])
. . . . | |    {map_select}
. . . . | |    -3
. . . . | |
. . . . | |\
. . . . | | 1. the_w : World
. . . . | | 2. e : E
. . . . | | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | | 4. ∀i:Id, t:ℕ.
. . . . | |    isnull(a(i;t))
. . . . | |    ⇒
. . . . | |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |    isrcv(l;a(i;t))
. . . . | |    ⇒
. . . . | |    destination(l) = i
. . . . | |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | | 6. ∀l:IdLnk, t:ℕ.
. . . . | |    ∃t':ℕ.
. . . . | |    t≤t'
. . . . | |    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | | 7. isrcv(kind(e))
. . . . | | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | | 9. destination(lnk(kind(e))) = loc(e)
. . . . | | 10. r : ℕ
. . . . | | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | | 12. ms : Msg
. . . . | | 13. msg(a(loc(e);time(e))) = ms
. . . . | | 14. l : IdLnk
. . . . | | 15. lnk(kind(e)) = l
. . . . | | 16. t' : ℕ
. . . . | | 17. time(e) = t'
. . . . | | 18. r<||snds(l;t')||
. . . . | | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | | 21. ||snds(l;m)||≤r
. . . . | | 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | | 23. snds(l;t')[r]
. . . . | |     =
. . . . | |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | | 24. m<t'
. . . . | | ⊢  m<||upto(t')||
. . . . | |   by RW0 Thm* ∀n:ℕ. ||upto(n)|| ∼ n    {length_upto} 0
. . . . | |
. . . . | .\
. . . . | . 1. the_w : World
. . . . | . 2. e : E
. . . . | . 3. ∀i:Id, t:ℕ, l:IdLnk.
```

```
. . . . | . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . 4. ∀i:Id, t:ℕ.
. . . . | . |  isnull(a(i;t))
. . . . | . |  ⇒
. . . . | . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . |  isrcv(l;a(i;t))
. . . . | . |  ⇒
. . . . | . |  destination(l) = i
. . . . | . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . 6. ∀l:IdLnk, t:ℕ.
. . . . | . |  ∃t':ℕ.
. . . . | . |  t≤t'
. . . . | . |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . 7. isrcv(kind(e))
. . . . | . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . 10. r : ℕ
. . . . | . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . 12. ms : Msg
. . . . | . 13. msg(a(loc(e);time(e))) = ms
. . . . | . 14. l : IdLnk
. . . . | . 15. lnk(kind(e)) = l
. . . . | . 16. t' : ℕ
. . . . | . 17. time(e) = t'
. . . . | . 18. r<||snds(l;t')||
. . . . | . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . 21. ||snds(l;m)||≤r
. . . . | . 22. r-||snds(l;m)||<||(λt1.m(l;t1))(upto(t')[m])||
. . . . | . 23. snds(l;t')[r]
. . . . | . |   =
. . . . | . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . 24. m<t'
. . . . | . ⊢  (||snds(l;m)||≤₂r)∧₂
. . . . | . |  (r<₂||snds(l;m)||+||onlnk(l;m(source(l);m))||)
. . . . | . | by Reduce -3 THEN Subst' (upto(t')[m] = m) -3
. . . . | . |
. . . . | . |\
. . . . | . | 1. the_w : World
. . . . | . | 2. e : E
. . . . | . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . | 4. ∀i:Id, t:ℕ.
. . . . | . |    isnull(a(i;t))
. . . . | . |    ⇒
. . . . | . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . |    isrcv(l;a(i;t))
. . . . | . |    ⇒
. . . . | . |    destination(l) = i
. . . . | . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . | 6. ∀l:IdLnk, t:ℕ.
. . . . | . |    ∃t':ℕ.
. . . . | . |    t≤t'
```

```
. . . . | . |    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . | 7. isrcv(kind(e))
. . . . | . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . | 9. destination(lnk(kind(e))) = loc(e)
. . . . | . | 10. r : ℕ
. . . . | . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . | 12. ms : Msg
. . . . | . | 13. msg(a(loc(e);time(e))) = ms
. . . . | . | 14. l : IdLnk
. . . . | . | 15. lnk(kind(e)) = l
. . . . | . | 16. t' : ℕ
. . . . | . | 17. time(e) = t'
. . . . | . | 18. r<||snds(l;t')||
. . . . | . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . | 21. ||snds(l;m)||≤r
. . . . | . | 22. r-||snds(l;m)||<||m(l;upto(t')[m])||
. . . . | . | 23. snds(l;t')[r]
. . . . | . |     =
. . . . | . |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . | 24. m<t'
. . . . | . | ⊢  m<||upto(t')||
. . . . | . |   by RW0 Thm* ∀n:ℕ. ||upto(n)|| ∼ n    {length_upto} 0
. . . . | . |
. . . . | . |\
. . . . | . | 1. the_w : World
. . . . | . | 2. e : E
. . . . | . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . | 4. ∀i:Id, t:ℕ.
. . . . | . |    isnull(a(i;t))
. . . . | . |    ⇒
. . . . | . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . |    isrcv(l;a(i;t))
. . . . | . |    ⇒
. . . . | . |    destination(l) = i
. . . . | . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . | 6. ∀l:IdLnk, t:ℕ.
. . . . | . |    ∃t':ℕ.
. . . . | . |    t≤t'
. . . . | . |    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . | 7. isrcv(kind(e))
. . . . | . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . | 9. destination(lnk(kind(e))) = loc(e)
. . . . | . | 10. r : ℕ
. . . . | . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . | 12. ms : Msg
. . . . | . | 13. msg(a(loc(e);time(e))) = ms
. . . . | . | 14. l : IdLnk
. . . . | . | 15. lnk(kind(e)) = l
. . . . | . | 16. t' : ℕ
. . . . | . | 17. time(e) = t'
. . . . | . | 18. r<||snds(l;t')||
. . . . | . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
```

```
. . . . | . | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . | 21. ||snds(l;m)||≤r
. . . . | . | 22. r-||snds(l;m)||<||m(l;upto(t')[m])||
. . . . | . | 23. snds(l;t')[r]
. . . . | . |     =
. . . . | . |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . | 24. m<t'
. . . . | . | ⊢  upto(t')[m] = m
. . . . | . |   by BackThru
. . . . | . |      Thm* ∀m:ℕ, n:ℕm. upto(m)[n] = n    {select_upto}
. . . . | . |
. . . . | . .\
. . . . | . . 1. the_w : World
. . . . | . . 2. e : E
. . . . | . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . . 4. ∀i:Id, t:ℕ.
. . . . | . . |  isnull(a(i;t))
. . . . | . . |  ⇒
. . . . | . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . . |  isrcv(l;a(i;t))
. . . . | . . |  ⇒
. . . . | . . |  destination(l) = i
. . . . | . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . . 6. ∀l:IdLnk, t:ℕ.
. . . . | . . |  ∃t':ℕ.
. . . . | . . |  t≤t'
. . . . | . . |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . . 7. isrcv(kind(e))
. . . . | . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . . 10. r : ℕ
. . . . | . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . . 12. ms : Msg
. . . . | . . 13. msg(a(loc(e);time(e))) = ms
. . . . | . . 14. l : IdLnk
. . . . | . . 15. lnk(kind(e)) = l
. . . . | . . 16. t' : ℕ
. . . . | . . 17. time(e) = t'
. . . . | . . 18. r<||snds(l;t')||
. . . . | . . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . . 21. ||snds(l;m)||≤r
. . . . | . . 22. r-||snds(l;m)||<||m(l;m)||
. . . . | . . 23. snds(l;t')[r]
. . . . | . . |   =
. . . . | . . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . . 24. m<t'
. . . . | . . ⊢  (||snds(l;m)||≤₂r)∧₂
. . . . | . . |  (r<₂||snds(l;m)||+||onlnk(l;m(source(l);m))||)
. . . . | . . | by Unfold `w-ml` -3
. . . . | . . |
. . . . | . . 1. the_w : World
. . . . | . . 2. e : E
```

```
. . . . | . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . .    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . . 4. ∀i:Id, t:ℕ.
. . . . | . .    isnull(a(i;t))
. . . . | . .    ⇒
. . . . | . .    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . .    isrcv(l;a(i;t))
. . . . | . .    ⇒
. . . . | . .    destination(l) = i
. . . . | . .    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . . 6. ∀l:IdLnk, t:ℕ.
. . . . | . .    ∃t':ℕ.
. . . . | . .    t≤t'
. . . . | . .    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . . 7. isrcv(kind(e))
. . . . | . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . . 10. r : ℕ
. . . . | . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . . 12. ms : Msg
. . . . | . . 13. msg(a(loc(e);time(e))) = ms

. . . . | . . 14. l : IdLnk
. . . . | . . 15. lnk(kind(e)) = l
. . . . | . . 16. t' : ℕ
. . . . | . . 17. time(e) = t'
. . . . | . . 18. r<||snds(l;t')||
. . . . | . . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . . 21. ||snds(l;m)||≤r
. . . . | . . 22. r-||snds(l;m)||<||onlnk(l;m(source(l);m))||
. . . . | . . 23. snds(l;t')[r]
. . . . | . .     =
. . . . | . .     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . . 24. m<t'
. . . . | . . ⊢  (||snds(l;m)||≤₂r)∧₂
. . . . | . .    (r<₂||snds(l;m)||+||onlnk(l;m(source(l);m))||)
. . . . | . .   by AssertBY (||onlnk(l;m(source(l);m))|| ∈ ℤ)
. . . . | . .      (GenConclAtAddr [2;1;2] THEN Fold `w-Msg` -2
. . . . | . .       THEN
. . . . | . .       Unfold `w-Msg` -1)
. . . . | . .      THEN
. . . . | . .      RW assert_pushdownC 0
. . . . |
. . . . |\
. . . . | 1. the_w : World
. . . . | 2. e : E
. . . . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | 4. ∀i:Id, t:ℕ.
. . . . | |  isnull(a(i;t))
. . . . | |  ⇒
. . . . | |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |  isrcv(l;a(i;t))
```

```
. . . . | |  ⇒
. . . . | |  destination(l) = i
. . . . | |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | 6. ∀l:IdLnk, t:ℕ.
. . . . | |  ∃t':ℕ.
. . . . | |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | 7. isrcv(kind(e))
. . . . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | 9. destination(lnk(kind(e))) = loc(e)
. . . . | 10. r : ℕ
. . . . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | 12. ms : Msg
. . . . | 13. msg(a(loc(e);time(e))) = ms
. . . . | 14. l : IdLnk
. . . . | 15. lnk(kind(e)) = l
. . . . | 16. t' : ℕ
. . . . | 17. time(e) = t'
. . . . | 18. r<||snds(l;t')||
. . . . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | 21. ||snds(l;m)||≤r
. . . . | 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | 23. snds(l;t')[r]
. . . . | |   =
. . . . | |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | 24. m<t'
. . . . | ⊢  onlnk(l;m(source(l);m))[(r-||snds(l;m)||)] = ms
. . . . | | by Assert (snds(l;t')[r] = ms)
. . . . | |
. . . . | |\
. . . . | | .....assertion .....
. . . . | | 1. the_w : World
. . . . | | 2. e : E
. . . . | | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | | 4. ∀i:Id, t:ℕ.
. . . . | |    isnull(a(i;t))
. . . . | |    ⇒
. . . . | |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |    isrcv(l;a(i;t))
. . . . | |    ⇒
. . . . | |    destination(l) = i
. . . . | |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | | 6. ∀l:IdLnk, t:ℕ.
. . . . | |    ∃t':ℕ.
. . . . | |    t≤t'
. . . . | |    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | | 7. isrcv(kind(e))
. . . . | | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | | 9. destination(lnk(kind(e))) = loc(e)
. . . . | | 10. r : ℕ
. . . . | | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | | 12. ms : Msg
. . . . | | 13. msg(a(loc(e);time(e))) = ms
```

```
. . . . | | 14. l : IdLnk
. . . . | | 15. lnk(kind(e)) = l
. . . . | | 16. t' : ℕ
. . . . | | 17. time(e) = t'
. . . . | | 18. r<||snds(l;t')||
. . . . | | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | | 21. ||snds(l;m)||≤r
. . . . | | 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | | 23. snds(l;t')[r]
. . . . | |     =
. . . . | |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | | 24. m<t'
. . . . | | ⊢  snds(l;t')[r] = ms  by Unfold `w-snds` 0 THEN Trivial
. . . . | |
. . . . | .\
. . . . | . 1. the_w : World
. . . . | . 2. e : E
. . . . | . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . 4. ∀i:Id, t:ℕ.
. . . . | . |  isnull(a(i;t))
. . . . | . |  ⇒
. . . . | . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . |  isrcv(l;a(i;t))
. . . . | . |  ⇒
. . . . | . |  destination(l) = i
. . . . | . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . 6. ∀l:IdLnk, t:ℕ.
. . . . | . |  ∃t':ℕ.
. . . . | . |  t≤t'
. . . . | . |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . 7. isrcv(kind(e))
. . . . | . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . 10. r : ℕ
. . . . | . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . 12. ms : Msg
. . . . | . 13. msg(a(loc(e);time(e))) = ms
. . . . | . 14. l : IdLnk
. . . . | . 15. lnk(kind(e)) = l
. . . . | . 16. t' : ℕ
. . . . | . 17. time(e) = t'
. . . . | . 18. r<||snds(l;t')||
. . . . | . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . 21. ||snds(l;m)||≤r
. . . . | . 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | . 23. snds(l;t')[r]
. . . . | . |   =
. . . . | . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . 24. m<t'
. . . . | . 25. snds(l;t')[r] = ms
. . . . | . ⊢  onlnk(l;m(source(l);m))[(r-||snds(l;m)||)] = ms
```

```
. . . . | . | by Assert
. . . . | . |    (onlnk(l;m(source(l);m))
. . . . | . |     =
. . . . | . |     map(λt1.m(l;t1);upto(t'))[m])
. . . . | . |
. . . . | . |\
. . . . | . | .....assertion .....
. . . . | . | 1. the_w : World
. . . . | . | 2. e : E
. . . . | . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . | 4. ∀i:Id, t:ℕ.
. . . . | . | |  isnull(a(i;t))
. . . . | . | |  ⇒
. . . . | . | |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | |  isrcv(l;a(i;t))
. . . . | . | |  ⇒
. . . . | . | |  destination(l) = i
. . . . | . | |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . | 6. ∀l:IdLnk, t:ℕ.
. . . . | . | |  ∃t':ℕ.
. . . . | . | |  t≤t'
. . . . | . | |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . | 7. isrcv(kind(e))
. . . . | . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . | 9. destination(lnk(kind(e))) = loc(e)
. . . . | . | 10. r : ℕ
. . . . | . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . | 12. ms : Msg
. . . . | . | 13. msg(a(loc(e);time(e))) = ms
. . . . | . | 14. l : IdLnk
. . . . | . | 15. lnk(kind(e)) = l
. . . . | . | 16. t' : ℕ
. . . . | . | 17. time(e) = t'
. . . . | . | 18. r<||snds(l;t')||
. . . . | . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . | 21. ||snds(l;m)||≤r
. . . . | . | 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | . | 23. snds(l;t')[r]
. . . . | . | |   =
. . . . | . | |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . | 24. m<t'
. . . . | . | 25. snds(l;t')[r] = ms
. . . . | . | ⊢  onlnk(l;m(source(l);m))
. . . . | . | |  =
. . . . | . | |  map(λt1.m(l;t1);upto(t'))[m]
. . . . | . | | by RWO
. . . . | . | |    Thm* ∀f:(A→B), as:A List, n:ℕ||as||.
. . . . | . | |         map(f;as)[n] = f(as[n])
. . . . | . | |    {map_select}
. . . . | . | |    0
. . . . | . | |    THEN
. . . . | . | |    Try (Unfold `w-Msg` 0 THEN Complete Auto)
```

```
. . . . | . | |
. . . . | . | |\
. . . . | . | | 1. the_w : World
. . . . | . | | 2. e : E
. . . . | . | | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . | | 4. ∀i:Id, t:ℕ.
. . . . | . | |    isnull(a(i;t))
. . . . | . | |    ⇒
. . . . | . | |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . | | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | |    isrcv(l;a(i;t))
. . . . | . | |    ⇒
. . . . | . | |    destination(l) = i
. . . . | . | |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . | | 6. ∀l:IdLnk, t:ℕ.
. . . . | . | |    ∃t':ℕ.
. . . . | . | |    t≤t'
. . . . | . | |    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . | | 7. isrcv(kind(e))
. . . . | . | | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . | | 9. destination(lnk(kind(e))) = loc(e)
. . . . | . | | 10. r : ℕ
. . . . | . | | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . | | 12. ms : Msg
. . . . | . | | 13. msg(a(loc(e);time(e))) = ms
. . . . | . | | 14. l : IdLnk
. . . . | . | | 15. lnk(kind(e)) = l
. . . . | . | | 16. t' : ℕ
. . . . | . | | 17. time(e) = t'
. . . . | . | | 18. r<||snds(l;t')||
. . . . | . | | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . | | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . | | 21. ||snds(l;m)||≤r
. . . . | . | | 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | . | | 23. snds(l;t')[r]
. . . . | . | |     =
. . . . | . | |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . | | 24. m<t'
. . . . | . | | 25. snds(l;t')[r] = ms
. . . . | . | | ⊢  m<||upto(t')||
. . . . | . | |   by RW0 Thm* ∀n:ℕ. ||upto(n)|| ∼ n    {length_upto}
. . . . | . | |      0
. . . . | . | |
. . . . | . | .\
. . . . | . | . 1. the_w : World
. . . . | . | . 2. e : E
. . . . | . | . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . | . 4. ∀i:Id, t:ℕ.
. . . . | . | . |  isnull(a(i;t))
. . . . | . | . |  ⇒
. . . . | . | . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . | . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | . |  isrcv(l;a(i;t))
```

```
. . . . | . | . |  ⇒
. . . . | . | . |  destination(l) = i
. . . . | . | . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . | . 6. ∀l:IdLnk, t:ℕ.
. . . . | . | . |  ∃t':ℕ.
. . . . | . | . |  t≤t'
. . . . | . | . |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . | . 7. isrcv(kind(e))
. . . . | . | . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . | . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . | . 10. r : ℕ
. . . . | . | . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . | . 12. ms : Msg
. . . . | . | . 13. msg(a(loc(e);time(e))) = ms
. . . . | . | . 14. l : IdLnk
. . . . | . | . 15. lnk(kind(e)) = l
. . . . | . | . 16. t' : ℕ
. . . . | . | . 17. time(e) = t'
. . . . | . | . 18. r<||snds(l;t')||
. . . . | . | . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . | . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . | . 21. ||snds(l;m)||≤r
. . . . | . | . 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | . | . 23. snds(l;t')[r]
. . . . | . | . |   =
. . . . | . | . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . | . 24. m<t'
. . . . | . | . 25. snds(l;t')[r] = ms
. . . . | . | . ⊢  onlnk(l;m(source(l);m))
. . . . | . | . |  =
. . . . | . | . |  (λt1.m(l;t1))(upto(t')[m])  by Reduce 0
. . . . | . | . |
. . . . | . | . 1. the_w : World
. . . . | . | . 2. e : E
. . . . | . | . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . | . 4. ∀i:Id, t:ℕ.
. . . . | . | . |  isnull(a(i;t))
. . . . | . | . |  ⇒
. . . . | . | . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . | . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | . |  isrcv(l;a(i;t))
. . . . | . | . |  ⇒
. . . . | . | . |  destination(l) = i
. . . . | . | . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . | . 6. ∀l:IdLnk, t:ℕ.
. . . . | . | . |  ∃t':ℕ.
. . . . | . | . |  t≤t'
. . . . | . | . |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . | . 7. isrcv(kind(e))
. . . . | . | . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . | . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . | . 10. r : ℕ
. . . . | . | . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . | . 12. ms : Msg
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. . . . | . | . 13. msg(a(loc(e);time(e))) = ms
. . . . | . | . 14. l : IdLnk
. . . . | . | . 15. lnk(kind(e)) = l
. . . . | . | . 16. t' : ℕ
. . . . | . | . 17. time(e) = t'
. . . . | . | . 18. r<||snds(l;t')||
. . . . | . | . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . | . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . | . 21. ||snds(l;m)||≤r
. . . . | . | . 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | . | . 23. snds(l;t')[r]
. . . . | . | . |   =
. . . . | . | . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . | . 24. m<t'
. . . . | . | . 25. snds(l;t')[r] = ms
. . . . | . | . ⊢  onlnk(l;m(source(l);m)) = m(l;upto(t')[m])
. . . . | . | . | by Unfold `w-ml` 0 THEN Subst' (upto(t')[m] = m) 0
. . . . | . | . |    THEN
. . . . | . | . |    Try (Unfold `w-Msg` 0 THEN Complete Auto)
. . . . | . | . |
. . . . | . | . |\
. . . . | . | . | 1. the_w : World
. . . . | . | . | 2. e : E
. . . . | . | . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | . |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . | . | 4. ∀i:Id, t:ℕ.
. . . . | . | . |    isnull(a(i;t))
. . . . | . | . |    ⇒
. . . . | . | . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . | . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | . |    isrcv(l;a(i;t))
. . . . | . | . |    ⇒
. . . . | . | . |    destination(l) = i
. . . . | . | . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . | . | 6. ∀l:IdLnk, t:ℕ.
. . . . | . | . |    ∃t':ℕ.
. . . . | . | . |    t≤t'
. . . . | . | . |    & isrcv(l;a(destination(l);t'))
. . . . | . | . |      ∨ queue(l;t') = nil
. . . . | . | . | 7. isrcv(kind(e))
. . . . | . | . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . | . | 9. destination(lnk(kind(e))) = loc(e)
. . . . | . | . | 10. r : ℕ
. . . . | . | . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . | . | 12. ms : Msg
. . . . | . | . | 13. msg(a(loc(e);time(e))) = ms
. . . . | . | . | 14. l : IdLnk
. . . . | . | . | 15. lnk(kind(e)) = l
. . . . | . | . | 16. t' : ℕ
. . . . | . | . | 17. time(e) = t'
. . . . | . | . | 18. r<||snds(l;t')||
. . . . | . | . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . | . | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . | . | 21. ||snds(l;m)||≤r
. . . . | . | . | 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
```

```
. . . . | . | . | 23. snds(l;t')[r]
. . . . | . | . |     =
. . . . | . | . |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . | . | 24. m<t'
. . . . | . | . | 25. snds(l;t')[r] = ms
. . . . | . | . | ⊢  m<||upto(t')||
. . . . | . | . |   by RW0 Thm* ∀n:ℕ. ||upto(n)|| ∼ n    {length_upto}
. . . . | . | . |      0
. . . . | . | . |
. . . . | . | . .\
. . . . | . | . . 1. the_w : World
. . . . | . | . . 2. e : E
. . . . | . | . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | . .    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . | . . 4. ∀i:Id, t:ℕ.
. . . . | . | . .    isnull(a(i;t))
. . . . | . | . .    ⇒
. . . . | . | . .    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . | . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . | . .    isrcv(l;a(i;t))
. . . . | . | . .    ⇒
. . . . | . | . .    destination(l) = i
. . . . | . | . .    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . | . . 6. ∀l:IdLnk, t:ℕ.
. . . . | . | . .    ∃t':ℕ.
. . . . | . | . .    t≤t'
. . . . | . | . .    & isrcv(l;a(destination(l);t'))
. . . . | . | . .      ∨ queue(l;t') = nil
. . . . | . | . . 7. isrcv(kind(e))
. . . . | . | . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . | . . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . | . . 10. r : ℕ
. . . . | . | . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . | . . 12. ms : Msg
. . . . | . | . . 13. msg(a(loc(e);time(e))) = ms
. . . . | . | . . 14. l : IdLnk
. . . . | . | . . 15. lnk(kind(e)) = l

. . . . | . | . . 16. t' : ℕ
. . . . | . | . . 17. time(e) = t'
. . . . | . | . . 18. r<||snds(l;t')||
. . . . | . | . . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . | . . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . | . . 21. ||snds(l;m)||≤r
. . . . | . | . . 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | . | . . 23. snds(l;t')[r]
. . . . | . | . .     =
. . . . | . | . .     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . | . . 24. m<t'
. . . . | . | . . 25. snds(l;t')[r] = ms
. . . . | . | . . ⊢  upto(t')[m] = m
. . . . | . | . .   by RW0
. . . . | . | . .      Thm* ∀m:ℕ, n:ℕm. upto(m)[n] = n
. . . . | . | . .      {select_upto}
. . . . | . | . .      0
. . . . | . |
```

```
. . . . | . .\
. . . . | . . 1. the_w : World
. . . . | . . 2. e : E
. . . . | . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . . 4. ∀i:Id, t:ℕ.
. . . . | . . |  isnull(a(i;t))
. . . . | . . |  ⇒
. . . . | . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . . |  isrcv(l;a(i;t))
. . . . | . . |  ⇒
. . . . | . . |  destination(l) = i
. . . . | . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . . 6. ∀l:IdLnk, t:ℕ.
. . . . | . . |  ∃t':ℕ.
. . . . | . . |  t≤t'
. . . . | . . |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . . 7. isrcv(kind(e))
. . . . | . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . . 10. r : ℕ
. . . . | . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . . 12. ms : Msg
. . . . | . . 13. msg(a(loc(e);time(e))) = ms
. . . . | . . 14. l : IdLnk
. . . . | . . 15. lnk(kind(e)) = l
. . . . | . . 16. t' : ℕ
. . . . | . . 17. time(e) = t'
. . . . | . . 18. r<||snds(l;t')||
. . . . | . . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . . 21. ||snds(l;m)||≤r
. . . . | . . 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | . . 23. snds(l;t')[r]
. . . . | . . |   =
. . . . | . . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . . 24. m<t'
. . . . | . . 25. snds(l;t')[r] = ms
. . . . | . . 26. onlnk(l;m(source(l);m))
. . . . | . . |   =
. . . . | . . |   map(λt1.m(l;t1);upto(t'))[m]
. . . . | . . ⊢  onlnk(l;m(source(l);m))[(r-||snds(l;m)||)] = ms
. . . . | . . | by Assert
. . . . | . . |    (onlnk(l;m(source(l);m))[(r-||snds(l;m)||)]
. . . . | . . |     =
. . . . | . . |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)])
. . . . | . . |
. . . . | . . .....assertion .....
. . . . | . . 1. the_w : World
. . . . | . . 2. e : E
. . . . | . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . . 4. ∀i:Id, t:ℕ.
. . . . | . . |  isnull(a(i;t))
```

```
. . . . | . . |  ⇒
. . . . | . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . . |  isrcv(l;a(i;t))
. . . . | . . |  ⇒
. . . . | . . |  destination(l) = i
. . . . | . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . . 6. ∀l:IdLnk, t:ℕ.
. . . . | . . |  ∃t':ℕ.
. . . . | . . |  t≤t'
. . . . | . . |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . . 7. isrcv(kind(e))
. . . . | . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . . 10. r : ℕ
. . . . | . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . . 12. ms : Msg
. . . . | . . 13. msg(a(loc(e);time(e))) = ms
. . . . | . . 14. l : IdLnk
. . . . | . . 15. lnk(kind(e)) = l
. . . . | . . 16. t' : ℕ
. . . . | . . 17. time(e) = t'
. . . . | . . 18. r<||snds(l;t')||
. . . . | . . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . . 21. ||snds(l;m)||≤r
. . . . | . . 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | . . 23. snds(l;t')[r]
. . . . | . . |   =
. . . . | . . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . . 24. m<t'
. . . . | . . 25. snds(l;t')[r] = ms
. . . . | . . 26. onlnk(l;m(source(l);m))
. . . . | . . |   =
. . . . | . . |   map(λt1.m(l;t1);upto(t'))[m]
. . . . | . . ⊢  onlnk(l;m(source(l);m))[(r-||snds(l;m)||)]
. . . . | . . |  =
. . . . | . . |  map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . . | by Analyze
. . . . | . . |
. . . . | . . 1. the_w : World
. . . . | . . 2. e : E
. . . . | . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . .    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . . 4. ∀i:Id, t:ℕ.
. . . . | . .    isnull(a(i;t))
. . . . | . .    ⇒
. . . . | . .    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | . .    isrcv(l;a(i;t))
. . . . | . .    ⇒
. . . . | . .    destination(l) = i
. . . . | . .    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . . 6. ∀l:IdLnk, t:ℕ.
. . . . | . .    ∃t':ℕ.
```

```
. . . . | . .    t≤t'
. . . . | . .    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . . 7. isrcv(kind(e))
. . . . | . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . . 10. r : ℕ
. . . . | . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . . 12. ms : Msg
. . . . | . . 13. msg(a(loc(e);time(e))) = ms
. . . . | . . 14. l : IdLnk
. . . . | . . 15. lnk(kind(e)) = l
. . . . | . . 16. t' : ℕ
. . . . | . . 17. time(e) = t'
. . . . | . . 18. r<||snds(l;t')||
. . . . | . . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | . . 21. ||snds(l;m)||≤r
. . . . | . . 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | . . 23. snds(l;t')[r]
. . . . | . .     =
. . . . | . .     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | . . 24. m<t'
. . . . | . . 25. snds(l;t')[r] = ms
. . . . | . . 26. onlnk(l;m(source(l);m))
. . . . | . .     =
. . . . | . .     map(λt1.m(l;t1);upto(t'))[m]
. . . . | . . ⊢  r-||snds(l;m)||<||onlnk(l;m(source(l);m))||
. . . . | . .   by HypSubst -1 0
. . . . |
. . . . |\
. . . . | .....subterm .....
. . . . | 1. the_w : World
. . . . | 2. e : E
. . . . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | 4. ∀i:Id, t:ℕ.
. . . . |    isnull(a(i;t))
. . . . |    ⇒
. . . . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |    isrcv(l;a(i;t))
. . . . |    ⇒
. . . . |    destination(l) = i
. . . . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | 6. ∀l:IdLnk, t:ℕ.
. . . . |    ∃t':ℕ.
. . . . |    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | 7. isrcv(kind(e))
. . . . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | 9. destination(lnk(kind(e))) = loc(e)
. . . . | 10. r : ℕ
. . . . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | 12. ms : Msg
. . . . | 13. msg(a(loc(e);time(e))) = ms
. . . . | 14. l : IdLnk
```

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. . . . | 15. lnk(kind(e)) = l
. . . . | 16. t' : ℕ
. . . . | 17. time(e) = t'
. . . . | 18. r<||snds(l;t')||
. . . . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | 21. ||snds(l;m)||≤r
. . . . | 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | 23. snds(l;t')[r]
. . . . |     =
. . . . |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | 24. m<t'
. . . . | 25. t : ℕt'
. . . . | ⊢  ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . |   by GenConclAtAddr [2;1;2] THEN Fold `w-Msg` -2
. . . . |      THEN
. . . . |      Unfold `w-Msg` -1

. . . . |
. . . . |\
. . . . | .....unproved Inclusion subgoal .....
. . . . | 1. the_w : World
. . . . | 2. e : E
. . . . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | 4. ∀i:Id, t:ℕ.
. . . . |    isnull(a(i;t))
. . . . |    ⇒
. . . . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |    isrcv(l;a(i;t))
. . . . |    ⇒
. . . . |    destination(l) = i
. . . . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | 6. ∀l:IdLnk, t:ℕ.
. . . . |    ∃t':ℕ.
. . . . |    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | 7. isrcv(kind(e))
. . . . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | 9. destination(lnk(kind(e))) = loc(e)
. . . . | 10. r : ℕ
. . . . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | 12. ms : Msg
. . . . | 13. msg(a(loc(e);time(e))) = ms
. . . . | 14. l : IdLnk
. . . . | 15. lnk(kind(e)) = l
. . . . | 16. t' : ℕ
. . . . | 17. time(e) = t'
. . . . | 18. r<||snds(l;t')||
. . . . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | 21. ||snds(l;m)||≤r
. . . . | 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | 23. snds(l;t')[r]
. . . . |     =
. . . . |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
```

```
. . . . | 24. m<t'
. . . . | 25. t : ℕt'
. . . . | 26. (||snds(l;t)||≤₂r)∧₂
. . . . |     (r<₂||snds(l;t)||+||onlnk(l;m(source(l);t))||)
. . . . | 27. m(source(l);t) = m(source(l);t)
. . . . | 28. {m:Msg(the_w.M)| source(mlnk(m)) = source(l) } List
. . . . | 29. {m:Msg(the_w.M)| source(mlnk(m)) = source(l) } List
. . . . | 30. u : {m:Msg(the_w.M)| source(mlnk(m)) = source(l) }
. . . . | ⊢  u ∈ Msg  by Unfold `w-Msg` 0
. . . . |
. . . . |\
. . . . | 1. the_w : World
. . . . | 2. e : E
. . . . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | 4. ∀i:Id, t:ℕ.
. . . . |    isnull(a(i;t))
. . . . |    ⇒
. . . . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |    isrcv(l;a(i;t))
. . . . |    ⇒
. . . . |    destination(l) = i
. . . . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | 6. ∀l:IdLnk, t:ℕ.
. . . . |    ∃t':ℕ.
. . . . |    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | 7. isrcv(kind(e))
. . . . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | 9. destination(lnk(kind(e))) = loc(e)
. . . . | 10. r : ℕ
. . . . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | 12. ms : Msg
. . . . | 13. msg(a(loc(e);time(e))) = ms
. . . . | 14. l : IdLnk
. . . . | 15. lnk(kind(e)) = l
. . . . | 16. t' : ℕ
. . . . | 17. time(e) = t'
. . . . | 18. r<||snds(l;t')||
. . . . | 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | 21. ||snds(l;m)||≤r
. . . . | 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | 23. snds(l;t')[r]
. . . . |     =
. . . . |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . | 24. m<t'
. . . . | 25. t : ℕt'
. . . . | 26. (||snds(l;t)||≤₂r)∧₂
. . . . |     (r<₂||snds(l;t)||+||onlnk(l;m(source(l);t))||)
. . . . | ⊢  0≤r-||snds(l;t)||
. . . . |   by AssertBY (||onlnk(l;m(source(l);t))|| ∈ ℤ)
. . . . |      (GenConclAtAddr [2;1;2] THEN Fold `w-Msg` -2
. . . . |       THEN
. . . . |       Unfold `w-Msg` -1)
```

```
. . . . |      THEN
. . . . |      RW assert_pushdownC -2
. . . . |
. . . . .\
. . . . . 1. the_w : World
. . . . . 2. e : E
. . . . . 3. ∀i:Id, t:ℕ, l:IdLnk.

. . . . .    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . . 4. ∀i:Id, t:ℕ.
. . . . .    isnull(a(i;t))
. . . . .    ⇒
. . . . .    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . .    isrcv(l;a(i;t))
. . . . .    ⇒
. . . . .    destination(l) = i
. . . . .    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . 6. ∀l:IdLnk, t:ℕ.
. . . . .    ∃t':ℕ.
. . . . .    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . 7. isrcv(kind(e))
. . . . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . 9. destination(lnk(kind(e))) = loc(e)
. . . . . 10. r : ℕ
. . . . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . . 12. ms : Msg
. . . . . 13. msg(a(loc(e);time(e))) = ms
. . . . . 14. l : IdLnk
. . . . . 15. lnk(kind(e)) = l
. . . . . 16. t' : ℕ
. . . . . 17. time(e) = t'
. . . . . 18. r<||snds(l;t')||
. . . . . 19. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . . 20. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . 21. ||snds(l;m)||≤r
. . . . . 22. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . . 23. snds(l;t')[r]
. . . . .     =
. . . . .     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . . 24. m<t'
. . . . . 25. t : ℕt'
. . . . . 26. (||snds(l;t)||≤₂r)∧₂
. . . . .     (r<₂||snds(l;t)||+||onlnk(l;m(source(l);t))||)
. . . . . ⊢  r-||snds(l;t)||<||onlnk(l;m(source(l);t))||
. . . . .   by AssertBY (||onlnk(l;m(source(l);t))|| ∈ ℤ)
. . . . .      (GenConclAtAddr [2;1;2] THEN Fold `w-Msg` -2
. . . . .       THEN
. . . . .       Unfold `w-Msg` -1)
. . . . .      THEN
. . . . .      RW assert_pushdownC -2
END
```