```
change-lemma  THM

⊢  ∀es:ES, x,i:Id, T:Type.
|  (∀x,y:T. Dec(x = y))
|  ⇒
|  (vartype(i;x) ⊆r T)
|  ⇒
|  (∀e',e:E.
|   e ≤ e'
|   ⇒
|   loc(e') = i
|   ⇒
|   ¬(x after e') = (x when e)
|   ⇒
|   (∃ev:E. e ≤ ev  & ev ≤ e'  & ¬(x after ev) = (x when ev)))
| by RepeatFor 8 (Analyze 0 THENA Complete Auto)
|    THEN
|    AssertBY (∀e:E. loc(e) = i ⇒ (x when e) ∈ T)
|    (Auto THEN DoSubsume THEN HypSubst -1 0)
|    THEN
|    AssertBY (∀e:E. loc(e) = i ⇒ (x after e) ∈ T)
|    (Auto THEN DoSubsume THEN HypSubst -1 0)
|
1. es : ES
2. x : Id
3. i : Id
4. T : Type
5. ∀x,y:T. Dec(x = y)
6. vartype(i;x) ⊆r T
7. e' : E
8. e : E
9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
⊢  e ≤ e'
|  ⇒
|  loc(e') = i
|  ⇒
|  ¬(x after e') = (x when e)
|  ⇒
|  (∃ev:E. e ≤ ev  & ev ≤ e'  & ¬(x after ev) = (x when ev))
| by Auto THEN Try BackThruSomeHyp
|
|\
| 1. es : ES
| 2. x : Id
| 3. i : Id
| 4. T : Type
| 5. ∀x,y:T. Dec(x = y)
| 6. vartype(i;x) ⊆r T
| 7. e' : E
| 8. e : E
| 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| 11. e ≤ e'
| 12. loc(e') = i
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| 13. ¬(x after e') = (x when e)
| ⊢  ∃ev:E. e ≤ ev  & ev ≤ e'  & ¬(x after ev) = (x when ev)
| | by Assert Dec((∃ev∈[e, e'].¬(x after ev) = (x when ev)))
| |
| |\
| | .....assertion .....
| | 1. es : ES
| | 2. x : Id
| | 3. i : Id
| | 4. T : Type
| | 5. ∀x,y:T. Dec(x = y)
| | 6. vartype(i;x) ⊆r T
| | 7. e' : E
| | 8. e : E
| | 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| | 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| | 11. e ≤ e'
| | 12. loc(e') = i
| | 13. ¬(x after e') = (x when e)
| | ⊢  Dec((∃ev∈[e, e'].¬(x after ev) = (x when ev)))
| |   by BackThru
| |      Thm* ∀F:(A→Prop), L:A List.
| |           (∀k:A. Dec(F(k))) ⇒ Dec((∃k∈L.F(k)))
| |      {decidable__l_exists}
| |      THEN
| |      Analyze -1
| |      THEN
| |      BackThruSomeHyp
| |
| .\
| . 1. es : ES
| . 2. x : Id
| . 3. i : Id
| . 4. T : Type
| . 5. ∀x,y:T. Dec(x = y)
| . 6. vartype(i;x) ⊆r T
| . 7. e' : E
| . 8. e : E
| . 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . 11. e ≤ e'
| . 12. loc(e') = i
| . 13. ¬(x after e') = (x when e)
| . 14. Dec((∃ev∈[e, e'].¬(x after ev) = (x when ev)))
| . ⊢  ∃ev:E. e ≤ ev  & ev ≤ e'  & ¬(x after ev) = (x when ev)
| . | by Analyze -1
| . |

| . |\
| . | 1. es : ES
| . | 2. x : Id
| . | 3. i : Id
| . | 4. T : Type
| . | 5. ∀x,y:T. Dec(x = y)
| . | 6. vartype(i;x) ⊆r T
| . | 7. e' : E
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| . | 8. e : E
| . | 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . | 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . | 11. e ≤ e'
| . | 12. loc(e') = i
| . | 13. ¬(x after e') = (x when e)
| . | 14. (∃ev∈[e, e'].¬(x after ev) = (x when ev))
| . | ⊢  ∃ev:E. e ≤ ev  & ev ≤ e'  & ¬(x after ev) = (x when ev)
| . | | by Unfold `l_exists` -1 THEN ParallelOp -1
| . | |
| . | |\
| . | | 1. es : ES
| . | | 2. x : Id
| . | | 3. i : Id
| . | | 4. T : Type
| . | | 5. ∀x,y:T. Dec(x = y)
| . | | 6. vartype(i;x) ⊆r T
| . | | 7. e' : E
| . | | 8. e : E
| . | | 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . | | 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . | | 11. e ≤ e'
| . | | 12. loc(e') = i
| . | | 13. ¬(x after e') = (x when e)
| . | | 14. ev : {e:E| loc(e) = i }
| . | | 15. (ev ∈ [e, e']) & ¬(x after ev) = (x when ev)
| . | | ⊢  e ≤ ev  & ev ≤ e'  & ¬(x after ev) = (x when ev)
| . | | | by ParallelOp -1
| . | | |
| . | | 1. es : ES
| . | | 2. x : Id
| . | | 3. i : Id
| . | | 4. T : Type
| . | | 5. ∀x,y:T. Dec(x = y)
| . | | 6. vartype(i;x) ⊆r T
| . | | 7. e' : E
| . | | 8. e : E
| . | | 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . | | 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . | | 11. e ≤ e'
| . | | 12. loc(e') = i
| . | | 13. ¬(x after e') = (x when e)
| . | | 14. ev : {e:E| loc(e) = i }
| . | | 15. (ev ∈ [e, e'])
| . | | ⊢  e ≤ ev  & ev ≤ e'   by RW0 "member-es-interval<" 0
| . | | |
| . | | 1. es : ES
| . | | 2. x : Id
| . | | 3. i : Id
| . | | 4. T : Type
| . | | 5. ∀x,y:T. Dec(x = y)
| . | | 6. vartype(i;x) ⊆r T
| . | | 7. e' : E
| . | | 8. e : E
| . | | 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
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| . | | 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . | | 11. e ≤ e'
| . | | 12. loc(e') = i
| . | | 13. ¬(x after e') = (x when e)
| . | | 14. ev : {e:E| loc(e) = i }
| . | | 15. (ev ∈ [e, e'])
| . | | ⊢  (ev ∈ [e, e'])
| . | |   by Using [`A',{e:E| loc(e) = i }]
| . | |      (BackThru
| . | |       Thm* ∀L:A List, x:A. (A ⊆r B) ⇒ (x ∈ L) ⇒ (x ∈ L)
| . | |       {l_member_subtype})
| . | |      THEN
| . | |      Analyze 0
| . | |
| . | |\
| . | | .....subterm .....
| . | | 1. es : ES
| . | | 2. x : Id
| . | | 3. i : Id
| . | | 4. T : Type
| . | | 5. ∀x,y:T. Dec(x = y)
| . | | 6. vartype(i;x) ⊆r T
| . | | 7. e' : E
| . | | 8. e : E
| . | | 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . | | 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . | | 11. e ≤ e'
| . | | 12. loc(e') = i
| . | | 13. ¬(x after e') = (x when e)
| . | | 14. ev : {e:E| loc(e) = i }
| . | | 15. (ev ∈ [e, e'])
| . | | 16. ¬(x after ev) = (x when ev)
| . | | 17. e1 : E
| . | | 18. e ≤ e1  & e1 ≤ e'
| . | | ⊢  (x after e1) ∈ T
| . | | | by Analyze -1 THEN Unfold `es-le` -1
| . | | |    THEN
| . | | |    Unfold `es-locl` -1
| . | | |    THEN
| . | | |    DoSubsume
| . | | |    THEN
| . | | |    Analyze -1
| . | | |
| . | | |\
| . | | | 1. es : ES
| . | | | 2. x : Id
| . | | | 3. i : Id
| . | | | 4. T : Type
| . | | | 5. ∀x,y:T. Dec(x = y)
| . | | | 6. vartype(i;x) ⊆r T
| . | | | 7. e' : E
| . | | | 8. e : E
| . | | | 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . | | | 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . | | | 11. e ≤ e'
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| . | | | 12. loc(e') = i
| . | | | 13. ¬(x after e') = (x when e)
| . | | | 14. ev : {e:E| loc(e) = i }
| . | | | 15. (ev ∈ [e, e'])
| . | | | 16. ¬(x after ev) = (x when ev)
| . | | | 17. e1 : E
| . | | | 18. e ≤ e1
| . | | | 19. loc(e1) = loc(e')
| . | | | 20. (e1 < e')
| . | | | ⊢  vartype(loc(e1);x) ⊆r T
| . | | |   by HypSubst -2 0 THEN Thin -2 THEN WeakSubstFor loc(e') 0
| . | | |
| . | | .\
| . | | . 1. es : ES
| . | | . 2. x : Id
| . | | . 3. i : Id
| . | | . 4. T : Type
| . | | . 5. ∀x,y:T. Dec(x = y)
| . | | . 6. vartype(i;x) ⊆r T
| . | | . 7. e' : E
| . | | . 8. e : E
| . | | . 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . | | . 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . | | . 11. e ≤ e'
| . | | . 12. loc(e') = i
| . | | . 13. ¬(x after e') = (x when e)
| . | | . 14. ev : {e:E| loc(e) = i }
| . | | . 15. (ev ∈ [e, e'])
| . | | . 16. ¬(x after ev) = (x when ev)
| . | | . 17. e1 : E
| . | | . 18. e ≤ e1
| . | | . 19. e1 = e'
| . | | . ⊢  vartype(loc(e1);x) ⊆r T
| . | | .   by HypSubst -1 0 THEN Thin -1 THEN WeakSubstFor loc(e') 0
| . | |
| . | .\
| . | . .....subterm .....
| . | . 1. es : ES
| . | . 2. x : Id
| . | . 3. i : Id
| . | . 4. T : Type
| . | . 5. ∀x,y:T. Dec(x = y)
| . | . 6. vartype(i;x) ⊆r T
| . | . 7. e' : E
| . | . 8. e : E
| . | . 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . | . 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . | . 11. e ≤ e'
| . | . 12. loc(e') = i
| . | . 13. ¬(x after e') = (x when e)
| . | . 14. ev : {e:E| loc(e) = i }
| . | . 15. (ev ∈ [e, e'])
| . | . 16. ¬(x after ev) = (x when ev)
| . | . 17. e1 : E
| . | . 18. e ≤ e1  & e1 ≤ e'
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| . | . ⊢  (x when e1) ∈ T
| . | . | by Analyze -1 THEN Unfold ‘es-le‘ -1
| . | . |    THEN
| . | . |    Unfold ‘es-locl‘ -1
| . | . |    THEN
| . | . |    DoSubsume
| . | . |    THEN
| . | . |    Analyze -1
| . | . |
| . | . |\
| . | . | 1. es : ES
| . | . | 2. x : Id
| . | . | 3. i : Id
| . | . | 4. T : Type
| . | . | 5. ∀x,y:T. Dec(x = y)
| . | . | 6. vartype(i;x) ⊆r T
| . | . | 7. e’ : E
| . | . | 8. e : E
| . | . | 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . | . | 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . | . | 11. e ≤ e’
| . | . | 12. loc(e’) = i
| . | . | 13. ¬(x after e’) = (x when e)
| . | . | 14. ev : {e:E| loc(e) = i }
| . | . | 15. (ev ∈ [e, e’])
| . | . | 16. ¬(x after ev) = (x when ev)
| . | . | 17. e1 : E
| . | . | 18. e ≤ e1
| . | . | 19. loc(e1) = loc(e’)
| . | . | 20. (e1 < e’)
| . | . | ⊢  vartype(loc(e1);x) ⊆r T
| . | . |   by HypSubst -2 0 THEN Thin -2 THEN WeakSubstFor loc(e’) 0
| . | . |
| . | . .\
| . | . . 1. es : ES
| . | . . 2. x : Id
| . | . . 3. i : Id
| . | . . 4. T : Type
| . | . . 5. ∀x,y:T. Dec(x = y)
| . | . . 6. vartype(i;x) ⊆r T
| . | . . 7. e’ : E
| . | . . 8. e : E
| . | . . 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . | . . 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . | . . 11. e ≤ e’
| . | . . 12. loc(e’) = i
| . | . . 13. ¬(x after e’) = (x when e)
| . | . . 14. ev : {e:E| loc(e) = i }
| . | . . 15. (ev ∈ [e, e’])
| . | . . 16. ¬(x after ev) = (x when ev)
| . | . . 17. e1 : E
| . | . . 18. e ≤ e1
| . | . . 19. e1 = e’
| . | . . ⊢  vartype(loc(e1);x) ⊆r T
| . | . .   by HypSubst -1 0 THEN Thin -1 THEN WeakSubstFor loc(e’) 0
```

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| . |
| . .\
| . . 1. es : ES
| . . 2. x : Id
| . . 3. i : Id
| . . 4. T : Type
| . . 5. ∀x,y:T. Dec(x = y)
| . . 6. vartype(i;x) ⊆r T
| . . 7. e' : E
| . . 8. e : E
| . . 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . 11. e ≤ e'
| . . 12. loc(e') = i
| . . 13. ¬(x after e') = (x when e)
| . . 14. ¬(∃ev∈[e, e'].¬(x after ev) = (x when ev))
| . . ⊢  ∃ev:E. e ≤ ev  & ev ≤ e'  & ¬(x after ev) = (x when ev)
| . . | by Analyze -2
| . . |
| . . 1. es : ES
| . . 2. x : Id
| . . 3. i : Id
| . . 4. T : Type
| . . 5. ∀x,y:T. Dec(x = y)
| . . 6. vartype(i;x) ⊆r T
| . . 7. e' : E
| . . 8. e : E
| . . 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . 11. e ≤ e'
| . . 12. loc(e') = i
| . . 13. ¬(∃ev∈[e, e'].¬(x after ev) = (x when ev))
| . . ⊢  (x after e') = (x when e)
| . . | by RepeatFor 3 (MoveToConcl -1)
| . . |    THEN
| . . |    RepeatFor 2 (MoveToConcl -3)
| . . |
| . . 1. es : ES
| . . 2. x : Id
| . . 3. i : Id
| . . 4. T : Type
| . . 5. ∀x,y:T. Dec(x = y)
| . . 6. vartype(i;x) ⊆r T
| . . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . ⊢  ∀e',e:E.
| . . |  e ≤ e'
| . . |  ⇒
| . . |  loc(e') = i
| . . |  ⇒
| . . |  ¬(∃ev∈[e, e'].¬(x after ev) = (x when ev))
| . . |  ⇒
| . . |  (x after e') = (x when e)
| . . | by LocLessInd
| . . |    THENA
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```
| . . |    (Auto THEN BackThruSomeHyp
| . . |     THEN
| . . |     Try (Analyze -1 THEN Complete Auto))
| . . |    THEN
| . . |    AllHyps
| . . |    (λh.
| . . |     Unfold `es-le` h THEN Analyze h
| . . |     THENL
| . . |     [Unfold `es-locl` h THEN Complete Auto;HypSubst h 0])
| . . |
| . . 1. es : ES
| . . 2. x : Id
| . . 3. i : Id
| . . 4. T : Type
| . . 5. ∀x,y:T. Dec(x = y)
| . . 6. vartype(i;x) ⊆r T
| . . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . 10. j : E
| . . 11. ∀k:E.
| . . |   (k <loc j)
| . . |   ⇒
| . . |   (∀e:E.
| . . |    e ≤ k
| . . |    ⇒
| . . |    loc(k) = i
| . . |    ⇒
| . . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . |    ⇒
| . . |    (x after k) = (x when e))
| . . ⊢  ∀e:E.
| . . |  e ≤ j
| . . |  ⇒
| . . |  loc(j) = i
| . . |  ⇒
| . . |  ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . |  ⇒
| . . |  (x after j) = (x when e)
| . . | by Auto THEN Try (Complete (BackThruSomeHyp THEN Analyze -1))
| . . |
| . . 1. es : ES
| . . 2. x : Id
| . . 3. i : Id
| . . 4. T : Type
| . . 5. ∀x,y:T. Dec(x = y)
| . . 6. vartype(i;x) ⊆r T
| . . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . 10. j : E
| . . 11. ∀k:E.
| . . |   (k <loc j)
| . . |   ⇒
| . . |   (∀e:E.
```

```
| . . |    e ≤ k
| . . |    ⇒
| . . |    loc(k) = i
| . . |    ⇒
| . . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . |    ⇒
| . . |    (x after k) = (x when e))
| . . 12. e : E
| . . 13. e ≤ j
| . . 14. loc(j) = i
| . . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . ⊢  (x after j) = (x when e)
| . . | by Decide ((x after j) = (x when j)) THENA BackThruSomeHyp
| . . |
| . . |\
| . . | 1. es : ES
| . . | 2. x : Id
| . . | 3. i : Id
| . . | 4. T : Type
| . . | 5. ∀x,y:T. Dec(x = y)
| . . | 6. vartype(i;x) ⊆r T
| . . | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | 10. j : E
| . . | 11. ∀k:E.
| . . | |   (k <loc j)
| . . | |   ⇒
| . . | |   (∀e:E.
| . . | |    e ≤ k
| . . | |    ⇒
| . . | |    loc(k) = i
| . . | |    ⇒
| . . | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | |    ⇒
| . . | |    (x after k) = (x when e))
| . . | 12. e : E
| . . | 13. e ≤ j
| . . | 14. loc(j) = i
| . . | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | 16. (x after j) = (x when j)
| . . | ⊢  (x after j) = (x when e)
| . . | | by HypSubst -1 0 THENA (Auto THEN BackThruSomeHyp)
| . . | |
| . . | |\
| . . | | 1. es : ES
| . . | | 2. x : Id
| . . | | 3. i : Id
| . . | | 4. T : Type
| . . | | 5. ∀x,y:T. Dec(x = y)
| . . | | 6. vartype(i;x) ⊆r T
| . . | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | 10. j : E
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```
| . . | | 11. ∀k:E.
| . . | | |   (k <loc j)
| . . | | |   ⇒
| . . | | |   (∀e:E.
| . . | | |    e ≤ k
| . . | | |    ⇒
| . . | | |    loc(k) = i
| . . | | |    ⇒
| . . | | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | |    ⇒
| . . | | |    (x after k) = (x when e))
| . . | | 12. e : E
| . . | | 13. e ≤ j
| . . | | 14. loc(j) = i
| . . | | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | 16. (x after j) = (x when j)
| . . | | ⊢  (x when j) = (x when e)  by Assert (loc(e) = i)
| . . | | |
| . . | | |\
| . . | | | .....assertion .....
| . . | | | 1. es : ES
| . . | | | 2. x : Id
| . . | | | 3. i : Id
| . . | | | 4. T : Type
| . . | | | 5. ∀x,y:T. Dec(x = y)
| . . | | | 6. vartype(i;x) ⊆r T
| . . | | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | | 10. j : E
| . . | | | 11. ∀k:E.
| . . | | |     (k <loc j)
| . . | | |     ⇒
| . . | | |     (∀e:E.
| . . | | |      e ≤ k
| . . | | |      ⇒
| . . | | |      loc(k) = i
| . . | | |      ⇒
| . . | | |      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | |      ⇒
| . . | | |      (x after k) = (x when e))
| . . | | | 12. e : E
| . . | | | 13. e ≤ j
| . . | | | 14. loc(j) = i
| . . | | | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | | 16. (x after j) = (x when j)
| . . | | | ⊢  loc(e) = i
| . . | | |   by FwdThru
| . . | | |      Thm* ∀es:ES, e,e':E. e ≤ e'  ⇒ loc(e) = loc(e') ∈ Id
| . . | | |      {es_DASH_le_DASH_loc}
| . . | | |      [-4]
| . . | | |
| . . | | .\
| . . | | . 1. es : ES
| . . | | . 2. x : Id
```

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| . . | | . 3. i : Id
| . . | | . 4. T : Type
| . . | | . 5. ∀x,y:T. Dec(x = y)
| . . | | . 6. vartype(i;x) ⊆r T
| . . | | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . 10. j : E
| . . | | . 11. ∀k:E.
| . . | | . |   (k <loc j)
| . . | | . |   ⇒
| . . | | . |   (∀e:E.
| . . | | . |    e ≤ k
| . . | | . |    ⇒
| . . | | . |    loc(k) = i
| . . | | . |    ⇒
| . . | | . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . |    ⇒
| . . | | . |    (x after k) = (x when e))
| . . | | . 12. e : E
| . . | | . 13. e ≤ j
| . . | | . 14. loc(j) = i
| . . | | . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . 16. (x after j) = (x when j)
| . . | | . 17. loc(e) = i
| . . | | . ⊢  (x when j) = (x when e)  by Decide first(j)
| . . | | . |
| . . | | . |\
| . . | | . | 1. es : ES
| . . | | . | 2. x : Id
| . . | | . | 3. i : Id
| . . | | . | 4. T : Type
| . . | | . | 5. ∀x,y:T. Dec(x = y)
| . . | | . | 6. vartype(i;x) ⊆r T
| . . | | . | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . | 10. j : E
| . . | | . | 11. ∀k:E.
| . . | | . | |   (k <loc j)
| . . | | . | |   ⇒
| . . | | . | |   (∀e:E.
| . . | | . | |    e ≤ k
| . . | | . | |    ⇒
| . . | | . | |    loc(k) = i
| . . | | . | |    ⇒
| . . | | . | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . | |    ⇒
| . . | | . | |    (x after k) = (x when e))
| . . | | . | 12. e : E
| . . | | . | 13. e ≤ j
| . . | | . | 14. loc(j) = i
| . . | | . | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . | 16. (x after j) = (x when j)
| . . | | . | 17. loc(e) = i
```

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| . . | | . | 18. first(j)
| . . | | . | ⊢  (x when j) = (x when e)  by Assert (j = e)
| . . | | . | |
| . . | | . | |\
| . . | | . | | .....assertion .....
| . . | | . | | 1. es : ES
| . . | | . | | 2. x : Id
| . . | | . | | 3. i : Id
| . . | | . | | 4. T : Type
| . . | | . | | 5. ∀x,y:T. Dec(x = y)
| . . | | . | | 6. vartype(i;x) ⊆r T
| . . | | . | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . | | 10. j : E
| . . | | . | | 11. ∀k:E.
| . . | | . | | |   (k <loc j)
| . . | | . | | |   ⇒
| . . | | . | | |   (∀e:E.
| . . | | . | | |    e ≤ k
| . . | | . | | |    ⇒
| . . | | . | | |    loc(k) = i
| . . | | . | | |    ⇒
| . . | | . | | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . | | |    ⇒
| . . | | . | | |    (x after k) = (x when e))
| . . | | . | | 12. e : E
| . . | | . | | 13. e ≤ j
| . . | | . | | 14. loc(j) = i
| . . | | . | | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . | | 16. (x after j) = (x when j)
| . . | | . | | 17. loc(e) = i
| . . | | . | | 18. first(j)
| . . | | . | | ⊢  j = e  by Unfold ‘es-le‘ -6 THEN Analyze -6
| . . | | . | | |
| . . | | . | | 1. es : ES
| . . | | . | | 2. x : Id
| . . | | . | | 3. i : Id
| . . | | . | | 4. T : Type
| . . | | . | | 5. ∀x,y:T. Dec(x = y)
| . . | | . | | 6. vartype(i;x) ⊆r T
| . . | | . | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . | | 10. j : E
| . . | | . | | 11. ∀k:E.
| . . | | . | |     (k <loc j)
| . . | | . | |     ⇒
| . . | | . | |     (∀e:E.
| . . | | . | |      e ≤ k
| . . | | . | |      ⇒
| . . | | . | |      loc(k) = i
| . . | | . | |      ⇒
| . . | | . | |      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . | |      ⇒
```

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| . . | | . | |      (x after k) = (x when e))
| . . | | . | | 12. e : E
| . . | | . | | 13. (e <loc j)
| . . | | . | | 14. loc(j) = i
| . . | | . | | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . | | 16. (x after j) = (x when j)
| . . | | . | | 17. loc(e) = i
| . . | | . | | 18. first(j)
| . . | | . | | ⊢  j = e
| . . | | . | |   by RW0
| . . | | . | |      Thm* ∀the_es:ES, e,e':E.
| . . | | . | |           (e <loc e')
| . . | | . | |           ⟺
| . . | | . | |           ¬first(e')
| . . | | . | |           & e = pred(e') ∈ E ∨ (e <loc pred(e'))
| . . | | . | |      {es_DASH_locl_DASH_iff}
| . . | | . | |      -6
| . . | | . | |
| . . | | . | .\
| . . | | . | . 1. es : ES
| . . | | . | . 2. x : Id
| . . | | . | . 3. i : Id
| . . | | . | . 4. T : Type
| . . | | . | . 5. ∀x,y:T. Dec(x = y)
| . . | | . | . 6. vartype(i;x) ⊆r T
| . . | | . | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . | . 10. j : E
| . . | | . | . 11. ∀k:E.
| . . | | . | . |   (k <loc j)
| . . | | . | . |   ⇒
| . . | | . | . |   (∀e:E.
| . . | | . | . |    e ≤ k
| . . | | . | . |    ⇒
| . . | | . | . |    loc(k) = i
| . . | | . | . |    ⇒
| . . | | . | . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . | . |    ⇒
| . . | | . | . |    (x after k) = (x when e))
| . . | | . | . 12. e : E
| . . | | . | . 13. e ≤ j
| . . | | . | . 14. loc(j) = i
| . . | | . | . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . | . 16. (x after j) = (x when j)
| . . | | . | . 17. loc(e) = i
| . . | | . | . 18. first(j)
| . . | | . | . 19. j = e
| . . | | . | . ⊢  (x when j) = (x when e)
| . . | | . | . | by StrongHypSubst -1 0
| . . | | . | . |    THEN
| . . | | . | . |    Try (BackThruSomeHyp THEN Analyze -1)
| . . | | . | . |
| . . | | . | . 1. es : ES
| . . | | . | . 2. x : Id
```

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| . . | | . | . 3. i : Id
| . . | | . | . 4. T : Type
| . . | | . | . 5. ∀x,y:T. Dec(x = y)
| . . | | . | . 6. vartype(i;x) ⊆r T
| . . | | . | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . | . 10. j : E
| . . | | . | . 11. ∀k:E.
| . . | | . | .     (k <loc j)
| . . | | . | .     ⇒
| . . | | . | .     (∀e:E.
| . . | | . | .      e ≤ k
| . . | | . | .      ⇒
| . . | | . | .      loc(k) = i
| . . | | . | .      ⇒
| . . | | . | .      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . | .      ⇒
| . . | | . | .      (x after k) = (x when e))
| . . | | . | . 12. e : E
| . . | | . | . 13. e ≤ j
| . . | | . | . 14. loc(j) = i
| . . | | . | . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . | . 16. (x after j) = (x when j)
| . . | | . | . 17. loc(e) = i
| . . | | . | . 18. first(j)
| . . | | . | . 19. j = e
| . . | | . | . 20. z : E
| . . | | . | . 21. z = e
| . . | | . | . ⊢  loc(z) = i  by HypSubst -1 0
| . . | | . |
| . . | | . .\
| . . | | . . 1. es : ES
| . . | | . . 2. x : Id
| . . | | . . 3. i : Id
| . . | | . . 4. T : Type
| . . | | . . 5. ∀x,y:T. Dec(x = y)
| . . | | . . 6. vartype(i;x) ⊆r T
| . . | | . . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . 10. j : E
| . . | | . . 11. ∀k:E.
| . . | | . . |   (k <loc j)
| . . | | . . |   ⇒
| . . | | . . |   (∀e:E.
| . . | | . . |    e ≤ k
| . . | | . . |    ⇒
| . . | | . . |    loc(k) = i
| . . | | . . |    ⇒
| . . | | . . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . |    ⇒
| . . | | . . |    (x after k) = (x when e))
| . . | | . . 12. e : E
| . . | | . . 13. e ≤ j
```

```
| . . | | . . 14. loc(j) = i
| . . | | . . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . 16. (x after j) = (x when j)
| . . | | . . 17. loc(e) = i
| . . | | . . 18. ¬first(j)
| . . | | . . ⊢  (x when j) = (x when e)  by Inst_ES_Axiom [j;x] 6
| . . | | . . |
| . . | | . . 1. es : ES
| . . | | . . 2. x : Id
| . . | | . . 3. i : Id
| . . | | . . 4. T : Type
| . . | | . . 5. ∀x,y:T. Dec(x = y)
| . . | | . . 6. vartype(i;x) ⊆r T
| . . | | . . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . 10. j : E
| . . | | . . 11. ∀k:E.
| . . | | . . |   (k <loc j)
| . . | | . . |   ⇒
| . . | | . . |   (∀e:E.
| . . | | . . |    e ≤ k
| . . | | . . |    ⇒
| . . | | . . |    loc(k) = i
| . . | | . . |    ⇒
| . . | | . . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . |    ⇒
| . . | | . . |    (x after k) = (x when e))
| . . | | . . 12. e : E
| . . | | . . 13. e ≤ j
| . . | | . . 14. loc(j) = i
| . . | | . . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . 16. (x after j) = (x when j)
| . . | | . . 17. loc(e) = i
| . . | | . . 18. ¬first(j)
| . . | | . . 19. (x when j) = (x after pred(j))
| . . | | . . ⊢  (x when j) = (x when e)
| . . | | . . | by Unfold ‘es-le‘ -7 THEN Analyze -7
| . . | | . . |
| . . | | . . |\
| . . | | . . | 1. es : ES
| . . | | . . | 2. x : Id
| . . | | . . | 3. i : Id
| . . | | . . | 4. T : Type
| . . | | . . | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | 6. vartype(i;x) ⊆r T
| . . | | . . | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | 10. j : E
| . . | | . . | 11. ∀k:E.
| . . | | . . | |   (k <loc j)
| . . | | . . | |   ⇒
| . . | | . . | |   (∀e:E.
| . . | | . . | |    e ≤ k
```

```
| . . | | . . | |    ⇒
| . . | | . . | |    loc(k) = i
| . . | | . . | |    ⇒
| . . | | . . | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | |    ⇒
| . . | | . . | |    (x after k) = (x when e))
| . . | | . . | 12. e : E
| . . | | . . | 13. (e <loc j)
| . . | | . . | 14. loc(j) = i
| . . | | . . | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | 16. (x after j) = (x when j)
| . . | | . . | 17. loc(e) = i
| . . | | . . | 18. ¬first(j)
| . . | | . . | 19. (x when j) = (x after pred(j))
| . . | | . . | ⊢  (x when j) = (x when e)  by InstHyp [pred(j);e] 11
| . . | | . . | |
| . . | | . . | |\
| . . | | . . | | .....antecedent .....
| . . | | . . | | 1. es : ES
| . . | | . . | | 2. x : Id
| . . | | . . | | 3. i : Id
| . . | | . . | | 4. T : Type
| . . | | . . | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | 6. vartype(i;x) ⊆r T
| . . | | . . | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | 10. j : E
| . . | | . . | | 11. ∀k:E.
| . . | | . . | |     (k <loc j)
| . . | | . . | |     ⇒
| . . | | . . | |     (∀e:E.
| . . | | . . | |      e ≤ k
| . . | | . . | |      ⇒
| . . | | . . | |      loc(k) = i
| . . | | . . | |      ⇒
| . . | | . . | |      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | |      ⇒
| . . | | . . | |      (x after k) = (x when e))
| . . | | . . | | 12. e : E
| . . | | . . | | 13. (e <loc j)
| . . | | . . | | 14. loc(j) = i
| . . | | . . | | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | | 16. (x after j) = (x when j)
| . . | | . . | | 17. loc(e) = i
| . . | | . . | | 18. ¬first(j)
| . . | | . . | | 19. (x when j) = (x after pred(j))
| . . | | . . | | ⊢  (pred(j) <loc j)
| . . | | . . | |   by BackThru
| . . | | . . | |      Thm* ∀the_es:ES, j:E.
| . . | | . . | |           ¬first(j) ⇒ (pred(j) <loc j)
| . . | | . . | |      {es_DASH_pred_DASH_locl}
| . . | | . . | |
| . . | | . . | |\
| . . | | . . | | .....antecedent .....
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| . . | | . . | | 1. es : ES
| . . | | . . | | 2. x : Id
| . . | | . . | | 3. i : Id
| . . | | . . | | 4. T : Type
| . . | | . . | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | 6. vartype(i;x) ⊆r T
| . . | | . . | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | 10. j : E
| . . | | . . | | 11. ∀k:E.
| . . | | . . | | |   (k <loc j)
| . . | | . . | | |   ⇒
| . . | | . . | | |   (∀e:E.
| . . | | . . | | |    e ≤ k
| . . | | . . | | |    ⇒
| . . | | . . | | |    loc(k) = i
| . . | | . . | | |    ⇒
| . . | | . . | | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | |    ⇒
| . . | | . . | | |    (x after k) = (x when e))
| . . | | . . | | 12. e : E

| . . | | . . | | 13. (e <loc j)
| . . | | . . | | 14. loc(j) = i
| . . | | . . | | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | | 16. (x after j) = (x when j)
| . . | | . . | | 17. loc(e) = i
| . . | | . . | | 18. ¬first(j)
| . . | | . . | | 19. (x when j) = (x after pred(j))
| . . | | . . | | ⊢  e ≤ pred(j)
| . . | | . . | | | by RW0
| . . | | . . | | |    Thm* ∀the_es:ES, e,e':E.
| . . | | . . | | |         (e <loc e')
| . . | | . . | | |         ⟺
| . . | | . . | | |         ¬first(e')
| . . | | . . | | |         & e = pred(e') ∈ E ∨ (e <loc pred(e'))
| . . | | . . | | |    {es_DASH_locl_DASH_iff}
| . . | | . . | | |    -7
| . . | | . . | | |

| . . | | . . | | 1. es : ES
| . . | | . . | | 2. x : Id
| . . | | . . | | 3. i : Id
| . . | | . . | | 4. T : Type
| . . | | . . | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | 6. vartype(i;x) ⊆r T
| . . | | . . | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | 10. j : E
| . . | | . . | | 11. ∀k:E.
| . . | | . . | |     (k <loc j)
| . . | | . . | |     ⇒
| . . | | . . | |     (∀e:E.
| . . | | . . | |      e ≤ k
| . . | | . . | |      ⇒
```

```
| . . | | . . | | |      loc(k) = i
| . . | | . . | | |      ⇒
| . . | | . . | | |      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | |      ⇒
| . . | | . . | | |      (x after k) = (x when e))
| . . | | . . | | | 12. e : E
| . . | | . . | | | 13. ¬first(j)
| . . | | . . | | | 14. e = pred(j) ∨ (e <loc pred(j))
| . . | | . . | | | 15. loc(j) = i
| . . | | . . | | | 16. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | | | 17. (x after j) = (x when j)
| . . | | . . | | | 18. loc(e) = i
| . . | | . . | | | 19. ¬first(j)
| . . | | . . | | | 20. (x when j) = (x after pred(j))
| . . | | . . | | | ⊢  e ≤ pred(j)
| . . | | . . | | |   by Unfold ‘es-le‘ 0 THEN SplitOrHyps
| . . | | . . | | |      THENL
| . . | | . . | | |      [OrRight;OrLeft]
| . . | | . . | | |
| . . | | . . | | |\
| . . | | . . | | | .....antecedent .....
| . . | | . . | | | 1. es : ES
| . . | | . . | | | 2. x : Id
| . . | | . . | | | 3. i : Id
| . . | | . . | | | 4. T : Type
| . . | | . . | | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | 6. vartype(i;x) ⊆r T
| . . | | . . | | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | 10. j : E
| . . | | . . | | | 11. ∀k:E.
| . . | | . . | | |     (k <loc j)
| . . | | . . | | |     ⇒
| . . | | . . | | |     (∀e:E.
| . . | | . . | | |      e ≤ k
| . . | | . . | | |      ⇒
| . . | | . . | | |      loc(k) = i
| . . | | . . | | |      ⇒
| . . | | . . | | |      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | |      ⇒
| . . | | . . | | |      (x after k) = (x when e))
| . . | | . . | | | 12. e : E
| . . | | . . | | | 13. (e <loc j)
| . . | | . . | | | 14. loc(j) = i
| . . | | . . | | | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | | | 16. (x after j) = (x when j)
| . . | | . . | | | 17. loc(e) = i
| . . | | . . | | | 18. ¬first(j)
| . . | | . . | | | 19. (x when j) = (x after pred(j))
| . . | | . . | | | ⊢  loc(pred(j)) = i
| . . | | . . | | |   by RW0
| . . | | . . | | |      Thm* ∀the_es:ES, e:E.
| . . | | . . | | |           ¬first(e) ⇒ loc(pred(e)) = loc(e) ∈ Id
| . . | | . . | | |      {es_DASH_loc_DASH_pred}
```

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| . . | | . . | |      0
| . . | | . . | |
| . . | | . . | |\
| . . | | . . | | .....antecedent .....
| . . | | . . | | 1. es : ES
| . . | | . . | | 2. x : Id
| . . | | . . | | 3. i : Id
| . . | | . . | | 4. T : Type
| . . | | . . | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | 6. vartype(i;x) ⊆r T
| . . | | . . | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | 10. j : E
| . . | | . . | | 11. ∀k:E.
| . . | | . . | | |   (k <loc j)
| . . | | . . | | |   ⇒
| . . | | . . | | |   (∀e:E.
| . . | | . . | | |    e ≤ k
| . . | | . . | | |    ⇒
| . . | | . . | | |    loc(k) = i
| . . | | . . | | |    ⇒
| . . | | . . | | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | |    ⇒
| . . | | . . | | |    (x after k) = (x when e))
| . . | | . . | | 12. e : E
| . . | | . . | | 13. (e <loc j)
| . . | | . . | | 14. loc(j) = i
| . . | | . . | | 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | | 16. (x after j) = (x when j)
| . . | | . . | | 17. loc(e) = i
| . . | | . . | | 18. ¬first(j)
| . . | | . . | | 19. (x when j) = (x after pred(j))
| . . | | . . | | ⊢  ¬(∃ev∈[e, pred(j)].¬(x after ev) = (x when ev))
| . . | | . . | | | by ParallelOp -5
| . . | | . . | | |    THEN
| . . | | . . | | |    Try (BackThruSomeHyp THEN Analyze -1)
| . . | | . . | | |
| . . | | . . | | |\
| . . | | . . | | | 1. es : ES
| . . | | . . | | | 2. x : Id
| . . | | . . | | | 3. i : Id
| . . | | . . | | | 4. T : Type
| . . | | . . | | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | 6. vartype(i;x) ⊆r T
| . . | | . . | | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | 10. j : E
| . . | | . . | | | 11. ∀k:E.
| . . | | . . | | | |   (k <loc j)
| . . | | . . | | | |   ⇒
| . . | | . . | | | |   (∀e:E.
| . . | | . . | | | |    e ≤ k
| . . | | . . | | | |    ⇒
```

```
| . . | | . . | | | |    loc(k) = i
| . . | | . . | | | |    ⇒
| . . | | . . | | | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | |    ⇒
| . . | | . . | | | |    (x after k) = (x when e))
| . . | | . . | | | 12. e : E
| . . | | . . | | | 13. (e <loc j)
| . . | | . . | | | 14. loc(j) = i
| . . | | . . | | | 15. (x after j) = (x when j)
| . . | | . . | | | 16. loc(e) = i
| . . | | . . | | | 17. ¬first(j)
| . . | | . . | | | 18. (x when j) = (x after pred(j))
| . . | | . . | | | 19. (∃ev∈[e, pred(j)].¬(x after ev) = (x when ev))
| . . | | . . | | | ⊢  (∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | | |  by RepeatFor 2 (ParallelOp -1)
| . . | | . . | | |    THEN
| . . | | . . | | |    Try (Analyze -2 THEN BackThruSomeHyp)
| . . | | . . | | |
| . . | | . . | | | 1. es : ES
| . . | | . . | | | 2. x : Id
| . . | | . . | | | 3. i : Id
| . . | | . . | | | 4. T : Type
| . . | | . . | | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | 6. vartype(i;x) ⊆r T
| . . | | . . | | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | 10. j : E
| . . | | . . | | | 11. ∀k:E.
| . . | | . . | | |    (k <loc j)
| . . | | . . | | |    ⇒
| . . | | . . | | |    (∀e:E.
| . . | | . . | | |     e ≤ k
| . . | | . . | | |     ⇒
| . . | | . . | | |     loc(k) = i
| . . | | . . | | |     ⇒
| . . | | . . | | |     ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | |     ⇒
| . . | | . . | | |     (x after k) = (x when e))
| . . | | . . | | | 12. e : E
| . . | | . . | | | 13. (e <loc j)
| . . | | . . | | | 14. loc(j) = i
| . . | | . . | | | 15. (x after j) = (x when j)
| . . | | . . | | | 16. loc(e) = i
| . . | | . . | | | 17. ¬first(j)
| . . | | . . | | | 18. (x when j) = (x after pred(j))
| . . | | . . | | | 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | 20. (ev ∈ [e, pred(j)])
| . . | | . . | | |    & ¬(x after ev) = (x when ev)
| . . | | . . | | | ⊢  (ev ∈ [e, j]) & ¬(x after ev) = (x when ev)
| . . | | . . | | |  by ParallelOp -1
| . . | | . . | | |
| . . | | . . | | | 1. es : ES
| . . | | . . | | | 2. x : Id
| . . | | . . | | | 3. i : Id
```

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| . . | | . . | | | 4. T : Type
| . . | | . . | | | 5. ∀x,y:T. Dec(x = y)

| . . | | . . | | | 6. vartype(i;x) ⊆r T
| . . | | . . | | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | 10. j : E
| . . | | . . | | | 11. ∀k:E.
| . . | | . . | | | |   (k <loc j)
| . . | | . . | | | |   ⇒
| . . | | . . | | | |   (∀e:E.
| . . | | . . | | | |    e ≤ k
| . . | | . . | | | |    ⇒
| . . | | . . | | | |    loc(k) = i
| . . | | . . | | | |    ⇒
| . . | | . . | | | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | |    ⇒
| . . | | . . | | | |    (x after k) = (x when e))
| . . | | . . | | | 12. e : E
| . . | | . . | | | 13. (e <loc j)
| . . | | . . | | | 14. loc(j) = i
| . . | | . . | | | 15. (x after j) = (x when j)
| . . | | . . | | | 16. loc(e) = i
| . . | | . . | | | 17. ¬first(j)
| . . | | . . | | | 18. (x when j) = (x after pred(j))
| . . | | . . | | | 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | 20. (ev ∈ [e, pred(j)])
| . . | | . . | | | ⊢  (ev ∈ [e, j])
| . . | | . . | | | | by Using [‘B’,E]
| . . | | . . | | | |    (BackThru
| . . | | . . | | | |     Thm* strong-subtype(A;B)
| . . | | . . | | | |          ⇒
| . . | | . . | | | |          (∀L:A List, x:B. (x ∈ L) ⇒ (x ∈ L))
| . . | | . . | | | |     {strong_DASH_subtype_DASH_l_member})
| . . | | . . | | | |
| . . | | . . | | | 1. es : ES
| . . | | . . | | | 2. x : Id
| . . | | . . | | | 3. i : Id
| . . | | . . | | | 4. T : Type
| . . | | . . | | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | 6. vartype(i;x) ⊆r T
| . . | | . . | | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | 10. j : E
| . . | | . . | | | 11. ∀k:E.
| . . | | . . | | | |   (k <loc j)
| . . | | . . | | | |   ⇒
| . . | | . . | | | |   (∀e:E.
| . . | | . . | | | |    e ≤ k
| . . | | . . | | | |    ⇒
| . . | | . . | | | |    loc(k) = i
| . . | | . . | | | |    ⇒
| . . | | . . | | | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | |    ⇒
```

```
| . . | | . . | | | |    (x after k) = (x when e))
| . . | | . . | | | 12. e : E
| . . | | . . | | | 13. (e <loc j)
| . . | | . . | | | 14. loc(j) = i
| . . | | . . | | | 15. (x after j) = (x when j)
| . . | | . . | | | 16. loc(e) = i
| . . | | . . | | | 17. ¬first(j)
| . . | | . . | | | 18. (x when j) = (x after pred(j))
| . . | | . . | | | 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | 20. (ev ∈ [e, pred(j)])
| . . | | . . | | | ⊢  (ev ∈ [e, j])
| . . | | . . | | | | by Using [‘B’,E]
| . . | | . . | | | |    (FwdThru
| . . | | . . | | | |     Thm* ∀L:A List, x:A.
| . . | | . . | | | |          (A ⊆r B) ⇒ (x ∈ L) ⇒ (x ∈ L)
| . . | | . . | | | |     {l_member_subtype}
| . . | | . . | | | |     [-1])
| . . | | . . | | | |
| . . | | . . | | | |\
| . . | | . . | | | | .....wf .....
| . . | | . . | | | | 1. es : ES
| . . | | . . | | | | 2. x : Id
| . . | | . . | | | | 3. i : Id
| . . | | . . | | | | 4. T : Type
| . . | | . . | | | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | | 6. vartype(i;x) ⊆r T
| . . | | . . | | | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | | 10. j : E
| . . | | . . | | | | 11. ∀k:E.
| . . | | . . | | | | |   (k <loc j)
| . . | | . . | | | | |   ⇒
| . . | | . . | | | | |   (∀e:E.
| . . | | . . | | | | |    e ≤ k
| . . | | . . | | | | |    ⇒
| . . | | . . | | | | |    loc(k) = i
| . . | | . . | | | | |    ⇒
| . . | | . . | | | | |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | | |    ⇒
| . . | | . . | | | | |    (x after k) = (x when e))
| . . | | . . | | | | 12. e : E
| . . | | . . | | | | 13. (e <loc j)
| . . | | . . | | | | 14. loc(j) = i
| . . | | . . | | | | 15. (x after j) = (x when j)
| . . | | . . | | | | 16. loc(e) = i
| . . | | . . | | | | 17. ¬first(j)

| . . | | . . | | | | 18. (x when j) = (x after pred(j))
| . . | | . . | | | | 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | | 20. (ev ∈ [e, pred(j)])
| . . | | . . | | | | ⊢  [e, pred(j)] ∈ {e:E| loc(e) = i } List
| . . | | . . | | | | | by Inst
| . . | | . . | | | | |    Thm* ∀es:ES, e,e’:E.
| . . | | . . | | | | |         [e, e’]
| . . | | . . | | | | |         ∈ {ev:E| loc(ev) = loc(e’) ∈ Id } List
```

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| . . | | . . | | | | |    {es_DASH_interval_wf2}
| . . | | . . | | | | |    [es;e;pred(j)]
| . . | | . . | | | | |    THEN
| . . | | . . | | | | |    RWO
| . . | | . . | | | | |    Thm* ∀the_es:ES, e:E.
| . . | | . . | | | | |         ¬first(e)
| . . | | . . | | | | |         ⇒
| . . | | . . | | | | |         loc(pred(e)) = loc(e) ∈ Id
| . . | | . . | | | | |    {es_DASH_loc_DASH_pred}
| . . | | . . | | | | |    -1
| . . | | . . | | | | |    THEN
| . . | | . . | | | | |    DoSubsume
| . . | | . . | | | | |
| . . | | . . | | | | .....subtype_rel .....
| . . | | . . | | | | 1. es : ES
| . . | | . . | | | | 2. x : Id
| . . | | . . | | | | 3. i : Id
| . . | | . . | | | | 4. T : Type
| . . | | . . | | | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | | 6. vartype(i;x) ⊆r T
| . . | | . . | | | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | | 10. j : E
| . . | | . . | | | | 11. ∀k:E.
| . . | | . . | | | |     (k <loc j)
| . . | | . . | | | |     ⇒
| . . | | . . | | | |     (∀e:E.
| . . | | . . | | | |      e ≤ k
| . . | | . . | | | |      ⇒
| . . | | . . | | | |      loc(k) = i
| . . | | . . | | | |      ⇒
| . . | | . . | | | |      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | |      ⇒
| . . | | . . | | | |      (x after k) = (x when e))
| . . | | . . | | | | 12. e : E
| . . | | . . | | | | 13. (e <loc j)
| . . | | . . | | | | 14. loc(j) = i
| . . | | . . | | | | 15. (x after j) = (x when j)
| . . | | . . | | | | 16. loc(e) = i
| . . | | . . | | | | 17. ¬first(j)
| . . | | . . | | | | 18. (x when j) = (x after pred(j))
| . . | | . . | | | | 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | | 20. (ev ∈ [e, pred(j)])
| . . | | . . | | | | 21. [e, pred(j)]
| . . | | . . | | | |     ∈ {ev:E| loc(ev) = loc(j) } List
| . . | | . . | | | | ⊢  ({ev:E
| . . | | . . | | | |      | loc(ev) = loc(j) } List) ⊆r ({e:E
| . . | | . . | | | |                                    | loc(e)
| . . | | . . | | | |                                      =
| . . | | . . | | | |                                      i } List)
| . . | | . . | | | |   by (HypSubst -7 0) THEN (Analyze 0)
| . . | | . . | | | |
| . . | | . . | | | |\
| . . | | . . | | | | .....antecedent .....
```

```
| . . | | . . | | | | 1. es : ES
| . . | | . . | | | | 2. x : Id
| . . | | . . | | | | 3. i : Id
| . . | | . . | | | | 4. T : Type
| . . | | . . | | | | 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | | 6. vartype(i;x) ⊆r T
| . . | | . . | | | | 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | | 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | | 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | | 10. j : E
| . . | | . . | | | | 11. ∀k:E.
| . . | | . . | | | |     (k <loc j)
| . . | | . . | | | |     ⇒
| . . | | . . | | | |     (∀e:E.
| . . | | . . | | | |      e ≤ k
| . . | | . . | | | |      ⇒
| . . | | . . | | | |      loc(k) = i
| . . | | . . | | | |      ⇒
| . . | | . . | | | |      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | |      ⇒
| . . | | . . | | | |      (x after k) = (x when e))
| . . | | . . | | | | 12. e : E
| . . | | . . | | | | 13. (e <loc j)
| . . | | . . | | | | 14. loc(j) = i
| . . | | . . | | | | 15. (x after j) = (x when j)
| . . | | . . | | | | 16. loc(e) = i
| . . | | . . | | | | 17. ¬first(j)
| . . | | . . | | | | 18. (x when j) = (x after pred(j))
| . . | | . . | | | | 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | | 20. (ev ∈ [e, pred(j)])
| . . | | . . | | | | ⊢  {e:E| loc(e) = i } ⊆r E  by Analyze 0
| . . | | . . | | | |
| . . | | . . | | | .\
| . . | | . . | | | . 1. es : ES
| . . | | . . | | | . 2. x : Id
| . . | | . . | | | . 3. i : Id
| . . | | . . | | | . 4. T : Type
| . . | | . . | | | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | . 6. vartype(i;x) ⊆r T
| . . | | . . | | | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | . 10. j : E
| . . | | . . | | | . 11. ∀k:E.
| . . | | . . | | | . |   (k <loc j)
| . . | | . . | | | . |   ⇒
| . . | | . . | | | . |   (∀e:E.
| . . | | . . | | | . |    e ≤ k
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    loc(k) = i
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    (x after k) = (x when e))
| . . | | . . | | | . 12. e : E
```

```
| . . | | . . | | | . 13. (e <loc j)
| . . | | . . | | | . 14. loc(j) = i
| . . | | . . | | | . 15. (x after j) = (x when j)
| . . | | . . | | | . 16. loc(e) = i
| . . | | . . | | | . 17. ¬first(j)
| . . | | . . | | | . 18. (x when j) = (x after pred(j))
| . . | | . . | | | . 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | . 20. (ev ∈ [e, pred(j)])
| . . | | . . | | | . 21. (ev ∈ [e, pred(j)])
| . . | | . . | | | . ⊢  (ev ∈ [e, j])
| . . | | . . | | | . | by (RWO
| . . | | . . | | | . |     Thm* ∀es:ES, e,e',ev:E.
| . . | | . . | | | . |          (ev ∈ [e, e']) ⟺ e ≤ ev  & ev ≤ e'
| . . | | . . | | | . |     {member_DASH_es_DASH_interval}
| . . | | . . | | | . |     -1)
| . . | | . . | | | . |    THEN
| . . | | . . | | | . |    (RWO
| . . | | . . | | | . |     Thm* ∀es:ES, e,e',ev:E.
| . . | | . . | | | . |          (ev ∈ [e, e']) ⟺ e ≤ ev  & ev ≤ e'
| . . | | . . | | | . |     {member_DASH_es_DASH_interval}
| . . | | . . | | | . |     0)
| . . | | . . | | | . |
| . . | | . . | | | . 1. es : ES
| . . | | . . | | | . 2. x : Id
| . . | | . . | | | . 3. i : Id
| . . | | . . | | | . 4. T : Type
| . . | | . . | | | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | . 6. vartype(i;x) ⊆r T
| . . | | . . | | | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | . 10. j : E
| . . | | . . | | | . 11. ∀k:E.
| . . | | . . | | | . |   (k <loc j)
| . . | | . . | | | . |   ⇒
| . . | | . . | | | . |   (∀e:E.
| . . | | . . | | | . |    e ≤ k
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    loc(k) = i
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    (x after k) = (x when e))
| . . | | . . | | | . 12. e : E
| . . | | . . | | | . 13. (e <loc j)
| . . | | . . | | | . 14. loc(j) = i
| . . | | . . | | | . 15. (x after j) = (x when j)
| . . | | . . | | | . 16. loc(e) = i
| . . | | . . | | | . 17. ¬first(j)
| . . | | . . | | | . 18. (x when j) = (x after pred(j))
| . . | | . . | | | . 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | . 20. (ev ∈ [e, pred(j)])
| . . | | . . | | | . 21. e ≤ ev
| . . | | . . | | | . 22. ev ≤ pred(j)
| . . | | . . | | | . ⊢  ev ≤ j
```

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| . . | | . . | | | . | by Inst
| . . | | . . | | | . |    Thm* ∀the_es:ES. Trans x,y:E. x ≤ y
| . . | | . . | | | . |    {es_DASH_le_DASH_trans}
| . . | | . . | | | . |    [es]
| . . | | . . | | | . |
| . . | | . . | | | . 1. es : ES
| . . | | . . | | | . 2. x : Id
| . . | | . . | | | . 3. i : Id
| . . | | . . | | | . 4. T : Type
| . . | | . . | | | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | . 6. vartype(i;x) ⊆r T
| . . | | . . | | | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | . 10. j : E
| . . | | . . | | | . 11. ∀k:E.
| . . | | . . | | | . |   (k <loc j)
| . . | | . . | | | . |   ⇒
| . . | | . . | | | . |   (∀e:E.
| . . | | . . | | | . |    e ≤ k
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    loc(k) = i
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    (x after k) = (x when e))
| . . | | . . | | | . 12. e : E
| . . | | . . | | | . 13. (e <loc j)
| . . | | . . | | | . 14. loc(j) = i
| . . | | . . | | | . 15. (x after j) = (x when j)
| . . | | . . | | | . 16. loc(e) = i
| . . | | . . | | | . 17. ¬first(j)
| . . | | . . | | | . 18. (x when j) = (x after pred(j))
| . . | | . . | | | . 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | . 20. (ev ∈ [e, pred(j)])
| . . | | . . | | | . 21. e ≤ ev
| . . | | . . | | | . 22. ev ≤ pred(j)
| . . | | . . | | | . 23. Trans x,y:E. x ≤ y
| . . | | . . | | | . ⊢  ev ≤ j   by UseTrans pred(j)
| . . | | . . | | | . |
| . . | | . . | | | . 1. es : ES
| . . | | . . | | | . 2. x : Id
| . . | | . . | | | . 3. i : Id
| . . | | . . | | | . 4. T : Type
| . . | | . . | | | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | . 6. vartype(i;x) ⊆r T
| . . | | . . | | | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | . 10. j : E
| . . | | . . | | | . 11. ∀k:E.
| . . | | . . | | | . |   (k <loc j)
| . . | | . . | | | . |   ⇒
| . . | | . . | | | . |   (∀e:E.
| . . | | . . | | | . |    e ≤ k
```

```
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    loc(k) = i
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | . |    ⇒
| . . | | . . | | | . |    (x after k) = (x when e))
| . . | | . . | | | . 12. e : E
| . . | | . . | | | . 13. (e <loc j)
| . . | | . . | | | . 14. loc(j) = i
| . . | | . . | | | . 15. (x after j) = (x when j)
| . . | | . . | | | . 16. loc(e) = i
| . . | | . . | | | . 17. ¬first(j)
| . . | | . . | | | . 18. (x when j) = (x after pred(j))
| . . | | . . | | | . 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | . 20. (ev ∈ [e, pred(j)])
| . . | | . . | | | . 21. e ≤ ev
| . . | | . . | | | . 22. ev ≤ pred(j)
| . . | | . . | | | . 23. Trans x,y:E. x ≤ y
| . . | | . . | | | . ⊢  pred(j) ≤ j
| . . | | . . | | | . | by Unfold ‘es-le‘ 0 THEN OrLeft
| . . | | . . | | | . |
| . . | | . . | | | . 1. es : ES
| . . | | . . | | | . 2. x : Id
| . . | | . . | | | . 3. i : Id
| . . | | . . | | | . 4. T : Type
| . . | | . . | | | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | | . 6. vartype(i;x) ⊆r T
| . . | | . . | | | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | | . 10. j : E
| . . | | . . | | | . 11. ∀k:E.
| . . | | . . | | | .     (k <loc j)
| . . | | . . | | | .     ⇒
| . . | | . . | | | .     (∀e:E.
| . . | | . . | | | .      e ≤ k
| . . | | . . | | | .      ⇒
| . . | | . . | | | .      loc(k) = i
| . . | | . . | | | .      ⇒
| . . | | . . | | | .      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | | .      ⇒
| . . | | . . | | | .      (x after k) = (x when e))
| . . | | . . | | | . 12. e : E
| . . | | . . | | | . 13. (e <loc j)
| . . | | . . | | | . 14. loc(j) = i
| . . | | . . | | | . 15. (x after j) = (x when j)
| . . | | . . | | | . 16. loc(e) = i
| . . | | . . | | | . 17. ¬first(j)
| . . | | . . | | | . 18. (x when j) = (x after pred(j))
| . . | | . . | | | . 19. ev : {e:E| loc(e) = i }
| . . | | . . | | | . 20. (ev ∈ [e, pred(j)])
| . . | | . . | | | . 21. e ≤ ev
| . . | | . . | | | . 22. ev ≤ pred(j)
| . . | | . . | | | . 23. Trans x,y:E. x ≤ y
| . . | | . . | | | . ⊢  (pred(j) <loc j)
```

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| . . | | . . | | | .   by BackThru
| . . | | . . | | | .      Thm* ∀the_es:ES, j:E.
| . . | | . . | | | .           ¬first(j) ⇒ (pred(j) <loc j)
| . . | | . . | | | .      {es_DASH_pred_DASH_locl}
| . . | | . . | | |
| . . | | . . | | .\
| . . | | . . | | . .....aux .....
| . . | | . . | | . 1. es : ES
| . . | | . . | | . 2. x : Id
| . . | | . . | | . 3. i : Id
| . . | | . . | | . 4. T : Type
| . . | | . . | | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | . 6. vartype(i;x) ⊆r T
| . . | | . . | | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | . 10. j : E
| . . | | . . | | . 11. ∀k:E.
| . . | | . . | | . |   (k <loc j)
| . . | | . . | | . |   ⇒
| . . | | . . | | . |   (∀e:E.
| . . | | . . | | . |    e ≤ k
| . . | | . . | | . |    ⇒
| . . | | . . | | . |    loc(k) = i
| . . | | . . | | . |    ⇒
| . . | | . . | | . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | . |    ⇒
| . . | | . . | | . |    (x after k) = (x when e))
| . . | | . . | | . 12. e : E
| . . | | . . | | . 13. (e <loc j)
| . . | | . . | | . 14. loc(j) = i
| . . | | . . | | . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | | . 16. (x after j) = (x when j)
| . . | | . . | | . 17. loc(e) = i
| . . | | . . | | . 18. ¬first(j)
| . . | | . . | | . 19. (x when j) = (x after pred(j))
| . . | | . . | | . ⊢  [e, pred(j)] ∈ {e:E| loc(e) = i } List
| . . | | . . | | . | by Inst
| . . | | . . | | . |    Thm* ∀es:ES, e,e':E.
| . . | | . . | | . |         [e, e']
| . . | | . . | | . |         ∈ {ev:E| loc(ev) = loc(e') ∈ Id } List
| . . | | . . | | . |    {es_DASH_interval_wf2}
| . . | | . . | | . |    [es;e;pred(j)]
| . . | | . . | | . |    THEN
| . . | | . . | | . |    RWO
| . . | | . . | | . |    Thm* ∀the_es:ES, e:E.
| . . | | . . | | . |         ¬first(e) ⇒ loc(pred(e)) = loc(e) ∈ Id
| . . | | . . | | . |    {es_DASH_loc_DASH_pred}
| . . | | . . | | . |    -1
| . . | | . . | | . |    THEN
| . . | | . . | | . |    DoSubsume
| . . | | . . | | . |
| . . | | . . | | . .....subtype_rel .....
| . . | | . . | | . 1. es : ES
| . . | | . . | | . 2. x : Id
```

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| . . | | . . | | . 3. i : Id
| . . | | . . | | . 4. T : Type
| . . | | . . | | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | | . 6. vartype(i;x) ⊆r T
| . . | | . . | | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | | . 10. j : E
| . . | | . . | | . 11. ∀k:E.
| . . | | . . | | .     (k <loc j)
| . . | | . . | | .     ⇒
| . . | | . . | | .     (∀e:E.
| . . | | . . | | .      e ≤ k
| . . | | . . | | .      ⇒
| . . | | . . | | .      loc(k) = i
| . . | | . . | | .      ⇒
| . . | | . . | | .      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | | .      ⇒
| . . | | . . | | .      (x after k) = (x when e))
| . . | | . . | | . 12. e : E
| . . | | . . | | . 13. (e <loc j)
| . . | | . . | | . 14. loc(j) = i
| . . | | . . | | . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | | . 16. (x after j) = (x when j)
| . . | | . . | | . 17. loc(e) = i
| . . | | . . | | . 18. ¬first(j)
| . . | | . . | | . 19. (x when j) = (x after pred(j))
| . . | | . . | | . 20. [e, pred(j)] ∈ {ev:E| loc(ev) = loc(j) } List
| . . | | . . | | . ⊢  ({ev:E
| . . | | . . | | .       | loc(ev)
| . . | | . . | | .         =
| . . | | . . | | .         loc(j) } List) ⊆r ({e:E| loc(e) = i } List)
| . . | | . . | | .   by (HypSubst -7 0) THEN (Analyze 0)
| . . | | . . | |
| . . | | . . | .\
| . . | | . . | . 1. es : ES
| . . | | . . | . 2. x : Id
| . . | | . . | . 3. i : Id
| . . | | . . | . 4. T : Type
| . . | | . . | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | . 6. vartype(i;x) ⊆r T
| . . | | . . | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | . 10. j : E
| . . | | . . | . 11. ∀k:E.
| . . | | . . | . |   (k <loc j)
| . . | | . . | . |   ⇒
| . . | | . . | . |   (∀e:E.
| . . | | . . | . |    e ≤ k
| . . | | . . | . |    ⇒
| . . | | . . | . |    loc(k) = i
| . . | | . . | . |    ⇒
| . . | | . . | . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | . |    ⇒
```

```
| . . | | . . | . |    (x after k) = (x when e))
| . . | | . . | . 12. e : E
| . . | | . . | . 13. (e <loc j)
| . . | | . . | . 14. loc(j) = i
| . . | | . . | . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | . 16. (x after j) = (x when j)
| . . | | . . | . 17. loc(e) = i
| . . | | . . | . 18. ¬first(j)
| . . | | . . | . 19. (x when j) = (x after pred(j))
| . . | | . . | . 20. (x after pred(j)) = (x when e)
| . . | | . . | . ⊢  (x when j) = (x when e)  by Auto
| . . | | . . | . |
| . . | | . . | . 1. es : ES
| . . | | . . | . 2. x : Id
| . . | | . . | . 3. i : Id
| . . | | . . | . 4. T : Type
| . . | | . . | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | . 6. vartype(i;x) ⊆r T
| . . | | . . | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | . 10. j : E
| . . | | . . | . 11. ∀k:E.
| . . | | . . | . |   (k <loc j)
| . . | | . . | . |   ⇒
| . . | | . . | . |   (∀e:E.
| . . | | . . | . |    e ≤ k
| . . | | . . | . |    ⇒
| . . | | . . | . |    loc(k) = i
| . . | | . . | . |    ⇒
| . . | | . . | . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | . |    ⇒
| . . | | . . | . |    (x after k) = (x when e))
| . . | | . . | . 12. e : E
| . . | | . . | . 13. (e <loc j)
| . . | | . . | . 14. loc(j) = i
| . . | | . . | . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | . 16. (x after j) = (x when j)
| . . | | . . | . 17. loc(e) = i
| . . | | . . | . 18. ¬first(j)
| . . | | . . | . 19. (x when j) = (x after pred(j))
| . . | | . . | . 20. (x after pred(j)) = (x when e)
| . . | | . . | . ⊢  (x when j) = (x when e)  by HypSubst -2 0
| . . | | . . | . |
| . . | | . . | . .....subterm .....
| . . | | . . | . 1. es : ES
| . . | | . . | . 2. x : Id
| . . | | . . | . 3. i : Id
| . . | | . . | . 4. T : Type
| . . | | . . | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | . 6. vartype(i;x) ⊆r T
| . . | | . . | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | . 10. j : E
```

```
| . . | | . . | . 11. ∀k:E.
| . . | | . . | . |   (k <loc j)
| . . | | . . | . |   ⇒
| . . | | . . | . |   (∀e:E.
| . . | | . . | . |    e ≤ k
| . . | | . . | . |    ⇒
| . . | | . . | . |    loc(k) = i
| . . | | . . | . |    ⇒
| . . | | . . | . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | . |    ⇒
| . . | | . . | . |    (x after k) = (x when e))
| . . | | . . | . 12. e : E
| . . | | . . | . 13. (e <loc j)
| . . | | . . | . 14. loc(j) = i
| . . | | . . | . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | . 16. (x after j) = (x when j)
| . . | | . . | . 17. loc(e) = i
| . . | | . . | . 18. ¬first(j)
| . . | | . . | . 19. (x when j) = (x after pred(j))
| . . | | . . | . 20. (x after pred(j)) = (x when e)
| . . | | . . | . 21. z : vartype(loc(j);x)
| . . | | . . | . ⊢  z ∈ T  by DoSubsume
| . . | | . . | . |
| . . | | . . | . .....subtype_rel .....
| . . | | . . | . 1. es : ES
| . . | | . . | . 2. x : Id
| . . | | . . | . 3. i : Id
| . . | | . . | . 4. T : Type
| . . | | . . | . 5. ∀x,y:T. Dec(x = y)
| . . | | . . | . 6. vartype(i;x) ⊆r T
| . . | | . . | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . | . 10. j : E
| . . | | . . | . 11. ∀k:E.
| . . | | . . | .     (k <loc j)
| . . | | . . | .     ⇒
| . . | | . . | .     (∀e:E.
| . . | | . . | .      e ≤ k
| . . | | . . | .      ⇒
| . . | | . . | .      loc(k) = i
| . . | | . . | .      ⇒
| . . | | . . | .      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . | .      ⇒
| . . | | . . | .      (x after k) = (x when e))
| . . | | . . | . 12. e : E
| . . | | . . | . 13. (e <loc j)
| . . | | . . | . 14. loc(j) = i
| . . | | . . | . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . | . 16. (x after j) = (x when j)
| . . | | . . | . 17. loc(e) = i
| . . | | . . | . 18. ¬first(j)
| . . | | . . | . 19. (x when j) = (x after pred(j))
| . . | | . . | . 20. (x after pred(j)) = (x when e)
| . . | | . . | . 21. vartype(loc(j);x)
```

```
| . . | | . . | . ⊢  vartype(loc(j);x) ⊆r T  by WeakSubstFor loc(j) 0
| . . | | . . |
| . . | | . . .\
| . . | | . . . 1. es : ES
| . . | | . . . 2. x : Id
| . . | | . . . 3. i : Id
| . . | | . . . 4. T : Type
| . . | | . . . 5. ∀x,y:T. Dec(x = y)
| . . | | . . . 6. vartype(i;x) ⊆r T
| . . | | . . . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . . 10. j : E
| . . | | . . . 11. ∀k:E.
| . . | | . . . |   (k <loc j)
| . . | | . . . |   ⇒
| . . | | . . . |   (∀e:E.
| . . | | . . . |    e ≤ k
| . . | | . . . |    ⇒
| . . | | . . . |    loc(k) = i
| . . | | . . . |    ⇒
| . . | | . . . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . . |    ⇒
| . . | | . . . |    (x after k) = (x when e))
| . . | | . . . 12. e : E
| . . | | . . . 13. e = j
| . . | | . . . 14. loc(j) = i
| . . | | . . . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . . 16. (x after j) = (x when j)
| . . | | . . . 17. loc(e) = i
| . . | | . . . 18. ¬first(j)
| . . | | . . . 19. (x when j) = (x after pred(j))
| . . | | . . . ⊢  (x when j) = (x when e)
| . . | | . . . | by StrongHypSubst -7 0 THEN BackThruSomeHyp
| . . | | . . . |
| . . | | . . . 1. es : ES
| . . | | . . . 2. x : Id
| . . | | . . . 3. i : Id
| . . | | . . . 4. T : Type
| . . | | . . . 5. ∀x,y:T. Dec(x = y)
| . . | | . . . 6. vartype(i;x) ⊆r T
| . . | | . . . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | | . . . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | | . . . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | | . . . 10. j : E
| . . | | . . . 11. ∀k:E.
| . . | | . . .     (k <loc j)
| . . | | . . .     ⇒
| . . | | . . .     (∀e:E.
| . . | | . . .      e ≤ k
| . . | | . . .      ⇒
| . . | | . . .      loc(k) = i
| . . | | . . .      ⇒
| . . | | . . .      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | | . . .      ⇒
```

```
| . . | | . . .      (x after k) = (x when e))
| . . | | . . . 12. e : E
| . . | | . . . 13. e = j
| . . | | . . . 14. loc(j) = i
| . . | | . . . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | | . . . 16. (x after j) = (x when j)
| . . | | . . . 17. loc(e) = i
| . . | | . . . 18. ¬first(j)
| . . | | . . . 19. (x when j) = (x after pred(j))
| . . | | . . . 20. z : E
| . . | | . . . 21. z = j
| . . | | . . . ⊢  loc(z) = i  by HypSubst -1 0
| . . | |
| . . | .\
| . . | . 1. es : ES
| . . | . 2. x : Id
| . . | . 3. i : Id
| . . | . 4. T : Type
| . . | . 5. ∀x,y:T. Dec(x = y)
| . . | . 6. vartype(i;x) ⊆r T
| . . | . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . | . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . | . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . | . 10. j : E
| . . | . 11. ∀k:E.
| . . | .     (k <loc j)
| . . | .     ⇒
| . . | .     (∀e:E.
| . . | .      e ≤ k
| . . | .      ⇒
| . . | .      loc(k) = i
| . . | .      ⇒
| . . | .      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . | .      ⇒
| . . | .      (x after k) = (x when e))
| . . | . 12. e : E
| . . | . 13. e ≤ j
| . . | . 14. loc(j) = i
| . . | . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . | . 16. (x after j) = (x when j)
| . . | . 17. T
| . . | . ⊢  loc(e) = i
| . . | .   by FwdThru
| . . | .      Thm* ∀es:ES, e,e':E. e ≤ e'  ⇒ loc(e) = loc(e') ∈ Id
| . . | .      {es_DASH_le_DASH_loc}
| . . | .      [-5]
| . . |
| . . .\
| . . . 1. es : ES
| . . . 2. x : Id
| . . . 3. i : Id
| . . . 4. T : Type
| . . . 5. ∀x,y:T. Dec(x = y)
| . . . 6. vartype(i;x) ⊆r T
| . . . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
```

```
| . . . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . . 10. j : E
| . . . 11. ∀k:E.
| . . . |   (k <loc j)
| . . . |   ⇒
| . . . |   (∀e:E.
| . . . |    e ≤ k
| . . . |    ⇒
| . . . |    loc(k) = i
| . . . |    ⇒
| . . . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . . |    ⇒
| . . . |    (x after k) = (x when e))
| . . . 12. e : E
| . . . 13. e ≤ j
| . . . 14. loc(j) = i
| . . . 15. ¬(∃ev∈[e, j].¬(x after ev) = (x when ev))
| . . . 16. ¬(x after j) = (x when j)
| . . . ⊢  (x after j) = (x when e)
| . . . | by Analyze -2 THEN Unfold `l_exists` 0 THEN InstConcl [j]
| . . . |    THEN
| . . . |    Try (BackThruSomeHyp THEN Analyze -1)
| . . . |
| . . . 1. es : ES
| . . . 2. x : Id
| . . . 3. i : Id
| . . . 4. T : Type
| . . . 5. ∀x,y:T. Dec(x = y)
| . . . 6. vartype(i;x) ⊆r T
| . . . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . . 10. j : E
| . . . 11. ∀k:E.
| . . . |   (k <loc j)
| . . . |   ⇒
| . . . |   (∀e:E.
| . . . |    e ≤ k
| . . . |    ⇒
| . . . |    loc(k) = i
| . . . |    ⇒
| . . . |    ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . . |    ⇒
| . . . |    (x after k) = (x when e))
| . . . 12. e : E
| . . . 13. e ≤ j
| . . . 14. loc(j) = i
| . . . 15. ¬(x after j) = (x when j)
| . . . ⊢  (j ∈ [e, j])
| . . . | by Using [`B',E]
| . . . |    (BackThru
| . . . |     Thm* strong-subtype(A;B)
| . . . |          ⇒
| . . . |          (∀L:A List, x:B. (x ∈ L) ⇒ (x ∈ L))
```

```
| . . . |     {strong_DASH_subtype_DASH_1_member})
| . . . |    THEN
| . . . |    RWO
| . . . |    Thm* ∀es:ES, e,e',ev:E.
| . . . |         (ev ∈ [e, e']) ⟺ e ≤ ev  & ev ≤ e'
| . . . |    {member_DASH_es_DASH_interval}
| . . . |    0
| . . . |
| . . . 1. es : ES
| . . . 2. x : Id
| . . . 3. i : Id
| . . . 4. T : Type
| . . . 5. ∀x,y:T. Dec(x = y)
| . . . 6. vartype(i;x) ⊆r T
| . . . 7. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
| . . . 8. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
| . . . 9. WellFnd{i}(E;x,y.(x <loc y))
| . . . 10. j : E
| . . . 11. ∀k:E.
| . . .     (k <loc j)
| . . .     ⇒
| . . .     (∀e:E.
| . . .      e ≤ k
| . . .      ⇒
| . . .      loc(k) = i
| . . .      ⇒
| . . .      ¬(∃ev∈[e, k].¬(x after ev) = (x when ev))
| . . .      ⇒
| . . .      (x after k) = (x when e))
| . . . 12. e : E
| . . . 13. e ≤ j
| . . . 14. loc(j) = i
| . . . 15. ¬(x after j) = (x when j)
| . . . ⊢  j ≤ j   by Unfold `es-le` 0 THEN OrRight
|
.\
. 1. es : ES
. 2. x : Id
. 3. i : Id
. 4. T : Type
. 5. ∀x,y:T. Dec(x = y)
. 6. vartype(i;x) ⊆r T
. 7. e' : E
. 8. e : E
. 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
. 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
. 11. e ≤ e'
. 12. loc(e') = i
. ⊢  loc(e) = i  by Unfold `es-le` -2 THEN Analyze -2
. |
. |\
. | 1. es : ES
. | 2. x : Id
. | 3. i : Id
. | 4. T : Type
```

```
. | 5. ∀x,y:T. Dec(x = y)
. | 6. vartype(i;x) ⊆r T
. | 7. e' : E
. | 8. e : E
. | 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
. | 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
. | 11. (e <loc e')
. | 12. loc(e') = i
. | ⊢  loc(e) = i  by Unfold `es-locl` -2
. |
. .\
. . 1. es : ES
. . 2. x : Id
. . 3. i : Id
. . 4. T : Type
. . 5. ∀x,y:T. Dec(x = y)
. . 6. vartype(i;x) ⊆r T
. . 7. e' : E
. . 8. e : E
. . 9. ∀e:E. loc(e) = i ⇒ (x when e) ∈ T
. . 10. ∀e:E. loc(e) = i ⇒ (x after e) ∈ T
. . 11. e = e'
. . 12. loc(e') = i
. . ⊢  loc(e) = i  by HypSubst -2 0
END
```