```
w-match-exists  THM

⊢  ∀the_w:World, e:E.
|  FairFifo
|  ⇒
|  isrcv(kind(e)) ⇒ (∃t:ℕtime(e). match(lnk(kind(e));t;time(e)))
| by Auto THEN Unfold ‘fair-fifo‘ 3
|    THEN
|    Assert isrcv(lnk(kind(e));a(loc(e);time(e)))
|
|\
| .....assertion .....
| 1. the_w : World
| 2. e : E
| 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
| 4. ∀i:Id, t:ℕ.
| |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
| 5. ∀i:Id, t:ℕ, l:IdLnk.
| |  isrcv(l;a(i;t))
| |  ⇒
| |  destination(l) = i
| |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
| 6. ∀l:IdLnk, t:ℕ.
| |  ∃t’:ℕ. t≤t’ & isrcv(l;a(destination(l);t’)) ∨ queue(l;t’) = nil
| 7. isrcv(kind(e))
| ⊢  isrcv(lnk(kind(e));a(loc(e);time(e)))
| | by RepeatFor 4 (Thin 3) THEN Unfold ‘w-isrcvl‘ 0 THEN Analyze 2
| |    THEN
| |    Analyze 2
| |    THEN
| |    All (Unfolds [‘w-loc‘;‘w-time‘;‘w-ekind‘])
| |    THEN
| |    All Reduce
| |    THEN
| |    All (Unfolds [‘w-act‘])
| |    THEN
| |    All Reduce
| |
| 1. the_w : World
| 2. e1 : Id
| 3. e2 : ℕ
| 4. ¬isnull(a(e1;e2))  [not for witness]
| 5. isrcv(kind(a(e1;e2)))
| ⊢  ¬₂isnull(a(e1;e2))∧₂isrcv(kind(a(e1;e2)))∧₂
| |  lnk(kind(a(e1;e2))) = lnk(kind(a(e1;e2)))
| | by Unhide THEN AutoBoolCase isnull(a(e1;e2))
| |    THEN
| |    AutoBoolCase isrcv(kind(a(e1;e2)))
| |
| 1. the_w : World
| 2. e1 : Id
| 3. e2 : ℕ
| 4. ¬isnull(a(e1;e2))
| 5. isrcv(kind(a(e1;e2)))
| 6. ¬isnull(a(e1;e2))
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| 7. isrcv(kind(a(e1;e2)))
| ⊢  lnk(kind(a(e1;e2))) = lnk(kind(a(e1;e2)))
|   by RW assert_pushdownC 0
|
.\
. 1. the_w : World
. 2. e : E
. 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. 4. ∀i:Id, t:ℕ.
. |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. 5. ∀i:Id, t:ℕ, l:IdLnk.
. |  isrcv(l;a(i;t))
. |  ⇒
. |  destination(l) = i
. |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. 6. ∀l:IdLnk, t:ℕ.
. |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. 7. isrcv(kind(e))
. 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. ⊢  ∃t:ℕtime(e). match(lnk(kind(e));t;time(e))
. | by InstHyp [loc(e);time(e);lnk(kind(e))] 5
. |
. ...
. |
. |\
. | .....assertion .....
. | 1. the_w : World
. | 2. e : E
. | 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. | 4. ∀i:Id, t:ℕ.
. | |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. | 5. ∀i:Id, t:ℕ, l:IdLnk.
. | |  isrcv(l;a(i;t))
. | |  ⇒
. | |  destination(l) = i
. | |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. | 6. ∀l:IdLnk, t:ℕ.
. | |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. | 7. isrcv(kind(e))
. | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. | 9. destination(lnk(kind(e))) = loc(e)
. | 10. ||queue(lnk(kind(e));time(e))||≥1
. | |   & hd(queue(lnk(kind(e));time(e))) = msg(a(loc(e);time(e)))
. | ⊢  ||rcvs(lnk(kind(e));time(e))||<||snds(lnk(kind(e));time(e))||
. | | by ExRepD THEN Thin -1 THEN MoveToConcl -1
. | |    THEN
. | |    Unfold `w-queue` 0
. | |    THEN
. | |    GenConcl (||rcvs(lnk(kind(e));time(e))|| = r)
. | |    THEN
. | |    GenConcl (snds(lnk(kind(e));time(e)) = L)
. | |
. | |\
. | | 1. the_w : World
. | | 2. e : E
```

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. | | 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. | | 4. ∀i:Id, t:ℕ.
. | |    isnull(a(i;t))
. | |    ⇒
. | |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. | | 5. ∀i:Id, t:ℕ, l:IdLnk.
. | |    isrcv(l;a(i;t))
. | |    ⇒
. | |    destination(l) = i
. | |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. | | 6. ∀l:IdLnk, t:ℕ.
. | |    ∃t':ℕ.
. | |    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. | | 7. isrcv(kind(e))
. | | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. | | 9. destination(lnk(kind(e))) = loc(e)
. | | 10. L : Top List
. | | 11. snds(lnk(kind(e));time(e)) = L
. | | ⊢  0≤||rcvs(lnk(kind(e));time(e))||  by Auto
. | |
. | .\
. | . 1. the_w : World
. | . 2. e : E
. | . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. | . 4. ∀i:Id, t:ℕ.
. | .    isnull(a(i;t))
. | .    ⇒
. | .    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. | . 5. ∀i:Id, t:ℕ, l:IdLnk.
. | .    isrcv(l;a(i;t))
. | .    ⇒
. | .    destination(l) = i
. | .    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. | . 6. ∀l:IdLnk, t:ℕ.
. | .    ∃t':ℕ.
. | .    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. | . 7. isrcv(kind(e))
. | . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. | . 9. destination(lnk(kind(e))) = loc(e)
. | . 10. r : ℕ
. | . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. | . 12. L : Top List
. | . 13. snds(lnk(kind(e));time(e)) = L
. | . ⊢  ||nth_tl(r;L)||≥1 ⇒ r<||L||
. | .   by (RWO
. | .       Thm* ∀r:ℕ, L:Top List.
. | .             ||nth_tl(r;L)|| = if r<₂||L||→ ||L||-r else 0 fi
. | .       {general_length_nth_tl}
. | .       0)
. | .      THEN
. | .      SplitOnConclITE
. |
. .\
. . 1. the_w : World
. . 2. e : E
```

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. . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . 4. ∀i:Id, t:ℕ.
. . |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . |  isrcv(l;a(i;t))
. . |  ⇒
. . |  destination(l) = i
. . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . 6. ∀l:IdLnk, t:ℕ.
. . |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . 7. isrcv(kind(e))
. . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . 9. destination(lnk(kind(e))) = loc(e)
. . 10. ||queue(lnk(kind(e));time(e))||≥1
. . |    & hd(queue(lnk(kind(e));time(e))) = msg(a(loc(e);time(e)))
. . 11. ||rcvs(lnk(kind(e));time(e))||<||snds(lnk(kind(e));time(e))||
. . ⊢  ∃t:ℕtime(e). match(lnk(kind(e));t;time(e))
. . | by Unfold `w-queue` -2 THEN ExRepD THEN Thin -3
. . |     THEN
. . |     RWO
. . |     Thm* ∀m:ℕ, L:T List.
. . |          m<||L|| ⇒ (nth_tl(m;L) ~ [L[m] / nth_tl(1+m;L)])
. . |     {nth_tl_decomp}
. . |     -2
. . |     THEN
. . |     Reduce -2
. . |
. . 1. the_w : World
. . 2. e : E
. . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . 4. ∀i:Id, t:ℕ.
. . |  isnull(a(i;t)) ⇒ (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . |  isrcv(l;a(i;t))
. . |  ⇒
. . |  destination(l) = i
. . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . 6. ∀l:IdLnk, t:ℕ.
. . |  ∃t':ℕ. t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . 7. isrcv(kind(e))
. . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . 9. destination(lnk(kind(e))) = loc(e)
. . 10. snds(lnk(kind(e));time(e))[||rcvs(lnk(kind(e));time(e))||]
. . |    =
. . |    msg(a(loc(e);time(e)))
. . 11. ||rcvs(lnk(kind(e));time(e))||<||snds(lnk(kind(e));time(e))||
. . ⊢  ∃t:ℕtime(e). match(lnk(kind(e));t;time(e))
. . | by MoveToConcl -2 THEN MoveToConcl -1
. . |     THEN
. . |     GenConcl (||rcvs(lnk(kind(e));time(e))|| = r)
. . |     THEN
. . |     GenConcl (msg(a(loc(e);time(e))) = ms)
. . |
. . |\
. . | .....wf .....
```

```
. . | 1. the_w : World
. . | 2. e : E
. . | 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . | 4. ∀i:Id, t:ℕ.
. . |    isnull(a(i;t))
. . |    ⇒
. . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . |    isrcv(l;a(i;t))
. . |    ⇒
. . |    destination(l) = i
. . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . | 6. ∀l:IdLnk, t:ℕ.
. . |    ∃t':ℕ.
. . |    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . | 7. isrcv(kind(e))
. . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . | 9. destination(lnk(kind(e))) = loc(e)
. . | 10. r : ℕ
. . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . | ⊢  msg(a(loc(e);time(e))) ∈ Msg
. . |   by Using [`l',lnk(kind(e))] Auto
. . |
. . .\
. . . 1. the_w : World
. . . 2. e : E
. . . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . 4. ∀i:Id, t:ℕ.
. . . |  isnull(a(i;t))
. . . |  ⇒
. . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . |  isrcv(l;a(i;t))
. . . |  ⇒
. . . |  destination(l) = i
. . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . 6. ∀l:IdLnk, t:ℕ.
. . . |  ∃t':ℕ.
. . . |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . 7. isrcv(kind(e))
. . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . 9. destination(lnk(kind(e))) = loc(e)
. . . 10. r : ℕ
. . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . 12. ms : Msg
. . . 13. msg(a(loc(e);time(e))) = ms
. . . ⊢  r<||snds(lnk(kind(e));time(e))||
. . . |  ⇒
. . . |  snds(lnk(kind(e));time(e))[r] = ms
. . . |  ⇒
. . . |  (∃t:ℕtime(e). match(lnk(kind(e));t;time(e)))
. . . | by Unfold `w-match` 0 THEN HypSubst' -3 0
. . . |    THEN
. . . |    GenConcl (lnk(kind(e)) = l)
. . . |    THEN
```

```
. . . |    GenConcl (time(e) = t')
. . . |
. . . 1. the_w : World
. . . 2. e : E
. . . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . 4. ∀i:Id, t:ℕ.
. . . |  isnull(a(i;t))
. . . |  ⇒
. . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . |  isrcv(l;a(i;t))
. . . |  ⇒
. . . |  destination(l) = i
. . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . 6. ∀l:IdLnk, t:ℕ.
. . . |  ∃t':ℕ.
. . . |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . 7. isrcv(kind(e))
. . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . 9. destination(lnk(kind(e))) = loc(e)
. . . 10. r : ℕ
. . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . 12. ms : Msg
. . . 13. msg(a(loc(e);time(e))) = ms
. . . 14. l : IdLnk
. . . 15. lnk(kind(e)) = l
. . . 16. t' : ℕ
. . . 17. time(e) = t'
. . . ⊢  r<||snds(l;t')||
. . . |  ⇒
. . . |  snds(l;t')[r] = ms
. . . |  ⇒
. . . |  (∃t:ℕt'.
. . . |   (||snds(l;t)||≤₂r)∧₂
. . . |   (r<₂||snds(l;t)||+||onlnk(l;m(source(l);t))||))
. . . | by AssertBY (∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ)
. . . |    (Auto THEN GenConclAtAddr [2;1;2] THEN Fold `w-Msg` -2
. . . |     THEN
. . . |     Unfold `w-Msg` -1)
. . . |    THEN
. . . |    RW assert_pushdownC 0
. . . |    THEN
. . . |    Try BackThruSomeHyp
. . . |
. . . 1. the_w : World
. . . 2. e : E
. . . 3. ∀i:Id, t:ℕ, l:IdLnk. ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . 4. ∀i:Id, t:ℕ.
. . . |  isnull(a(i;t))
. . . |  ⇒
. . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . |  isrcv(l;a(i;t))
. . . |  ⇒
. . . |  destination(l) = i
```

```
. . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . 6. ∀l:IdLnk, t:ℕ.
. . . |  ∃t':ℕ.
. . . |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . 7. isrcv(kind(e))
. . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . 9. destination(lnk(kind(e))) = loc(e)
. . . 10. r : ℕ
. . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . 12. ms : Msg
. . . 13. msg(a(loc(e);time(e))) = ms
. . . 14. l : IdLnk
. . . 15. lnk(kind(e)) = l
. . . 16. t' : ℕ
. . . 17. time(e) = t'
. . . 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . 19. r<||snds(l;t')||
. . . 20. snds(l;t')[r] = ms
. . . ⊢  ∃t:ℕt'.
. . . |  ||snds(l;t)||≤r & r<||snds(l;t)||+||onlnk(l;m(source(l);t))||
. . . | by Unfold `w-snds` -1
. . . |    THEN
. . . |    Inst Thm: select_concat [Msg;map(λt1.m(l;t1);upto(t'));r]
. . . |
. . . |\
. . . | 1. the_w : World
. . . | 2. e : E
. . . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . | 4. ∀i:Id, t:ℕ.
. . . |    isnull(a(i;t))
. . . |    ⇒
. . . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . |    isrcv(l;a(i;t))
. . . |    ⇒
. . . |    destination(l) = i
. . . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . | 6. ∀l:IdLnk, t:ℕ.
. . . |    ∃t':ℕ.
. . . |    t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . | 7. isrcv(kind(e))
. . . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . | 9. destination(lnk(kind(e))) = loc(e)
. . . | 10. r : ℕ
. . . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . | 12. ms : Msg
. . . | 13. msg(a(loc(e);time(e))) = ms
. . . | 14. l : IdLnk
. . . | 15. lnk(kind(e)) = l
. . . | 16. t' : ℕ
. . . | 17. time(e) = t'
. . . | 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . | 19. r<||snds(l;t')||
. . . | 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
```

```
. . . | ⊢  r<||concat(map(λt1.m(l;t1);upto(t’)))||
. . . |   by Fold ‘w-snds‘ 0 THEN Trivial
. . . |
. . . .\
. . . . 1. the_w : World
. . . . 2. e : E
. . . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . 4. ∀i:Id, t:ℕ.
. . . . |  isnull(a(i;t))
. . . . |  ⇒
. . . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . |  isrcv(l;a(i;t))
. . . . |  ⇒
. . . . |  destination(l) = i
. . . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . 6. ∀l:IdLnk, t:ℕ.
. . . . |  ∃t’:ℕ.
. . . . |  t≤t’ & isrcv(l;a(destination(l);t’)) ∨ queue(l;t’) = nil
. . . . 7. isrcv(kind(e))
. . . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . 9. destination(lnk(kind(e))) = loc(e)
. . . . 10. r : ℕ
. . . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . 12. ms : Msg
. . . . 13. msg(a(loc(e);time(e))) = ms
. . . . 14. l : IdLnk
. . . . 15. lnk(kind(e)) = l
. . . . 16. t’ : ℕ
. . . . 17. time(e) = t’
. . . . 18. ∀t:ℕt’. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . 19. r<||snds(l;t’)||
. . . . 20. concat(map(λt1.m(l;t1);upto(t’)))[r] = ms
. . . . 21. ∃m:ℕ||map(λt1.m(l;t1);upto(t’))||.
. . . . |   ||concat(firstn(m;map(λt1.m(l;t1);upto(t’))))||≤r
. . . . |   & r-||concat(firstn(m;map(λt1.
. . . . |   m(l;t1);upto(t’))))||<||map(λt1.m(l;t1);upto(t’))[m]||
. . . . |   & concat(map(λt1.m(l;t1);upto(t’)))[r]
. . . . |     =
. . . . |     map(λt1.m(l;t1);upto(t’))[m][(r
. . . . |     -||concat(firstn(m;map(λt1.m(l;t1);upto(t’))))||)]
. . . . ⊢  ∃t:ℕt’.
. . . . |  ||snds(l;t)||≤r
. . . . |  & r<||snds(l;t)||+||onlnk(l;m(source(l);t))||
. . . . | by Analyze -1 THEN Assert (m<t’)
. . . . |
. . . . |\
. . . . | .....assertion .....
. . . . | 1. the_w : World
. . . . | 2. e : E
. . . . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | 4. ∀i:Id, t:ℕ.
. . . . | |  isnull(a(i;t))
```

```
. . . . | |  ⇒
. . . . | |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |  isrcv(l;a(i;t))
. . . . | |  ⇒
. . . . | |  destination(l) = i
. . . . | |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | 6. ∀l:IdLnk, t:ℕ.
. . . . | |  ∃t':ℕ.
. . . . | |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | 7. isrcv(kind(e))
. . . . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | 9. destination(lnk(kind(e))) = loc(e)
. . . . | 10. r : ℕ
. . . . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | 12. ms : Msg
. . . . | 13. msg(a(loc(e);time(e))) = ms
. . . . | 14. l : IdLnk
. . . . | 15. lnk(kind(e)) = l
. . . . | 16. t' : ℕ
. . . . | 17. time(e) = t'
. . . . | 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . | 19. r<||snds(l;t')||
. . . . | 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | 22. ||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||≤r
. . . . | |   & r-||concat(firstn(m;map(λt1.
. . . . | |   m(l;t1);upto(t'))))||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | |   & concat(map(λt1.m(l;t1);upto(t')))[r]
. . . . | |     =
. . . . | |     map(λt1.m(l;t1);upto(t'))[m][(r
. . . . | |     -||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||)]
. . . . | ⊢  m<t'  by Subst' (||map(λt1.m(l;t1);upto(t'))|| = t') -2
. . . . | |
. . . . | |\
. . . . | | 1. the_w : World
. . . . | | 2. e : E
. . . . | | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | | 4. ∀i:Id, t:ℕ.
. . . . | |    isnull(a(i;t))
. . . . | |    ⇒
. . . . | |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | |    isrcv(l;a(i;t))
. . . . | |    ⇒
. . . . | |    destination(l) = i
. . . . | |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | | 6. ∀l:IdLnk, t:ℕ.
. . . . | |    ∃t':ℕ.
. . . . | |    t≤t'
. . . . | |    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | | 7. isrcv(kind(e))
. . . . | | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | | 9. destination(lnk(kind(e))) = loc(e)
```

```
. . . . | | 10. r : ℕ
. . . . | | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | | 12. ms : Msg
. . . . | | 13. msg(a(loc(e);time(e))) = ms
. . . . | | 14. l : IdLnk
. . . . | | 15. lnk(kind(e)) = l
. . . . | | 16. t' : ℕ
. . . . | | 17. time(e) = t'
. . . . | | 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . | | 19. r<||snds(l;t')||
. . . . | | 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | | 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . | | 22. ||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||≤r
. . . . | |     & r-||concat(firstn(m;map(λt1.
. . . . | |     m(l;t1);upto(t'))))||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | |     & concat(map(λt1.m(l;t1);upto(t')))[r]
. . . . | |       =
. . . . | |       map(λt1.m(l;t1);upto(t'))[m][(r
. . . . | |       -||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||)]
. . . . | | ⊢  ||map(λt1.m(l;t1);upto(t'))|| = t'
. . . . | |   by RWO
. . . . | |      Thm* ∀f:(A→B), as:A List. ||map(f;as)|| = ||as|| ∈ ℤ
. . . . | |      {map_length}
. . . . | |      0
. . . . | |      THEN
. . . . | |      RWO Thm* ∀n:ℕ. ||upto(n)|| ~ n    {length_upto} 0
. . . . | |
. . . . | .\
. . . . | . 1. the_w : World
. . . . | . 2. e : E
. . . . | . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | .    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . | . 4. ∀i:Id, t:ℕ.
. . . . | .    isnull(a(i;t))
. . . . | .    ⇒
. . . . | .    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . | . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . | .    isrcv(l;a(i;t))
. . . . | .    ⇒
. . . . | .    destination(l) = i
. . . . | .    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . | . 6. ∀l:IdLnk, t:ℕ.
. . . . | .    ∃t':ℕ.
. . . . | .    t≤t'
. . . . | .    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . | . 7. isrcv(kind(e))
. . . . | . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . | . 9. destination(lnk(kind(e))) = loc(e)
. . . . | . 10. r : ℕ
. . . . | . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . | . 12. ms : Msg
. . . . | . 13. msg(a(loc(e);time(e))) = ms
. . . . | . 14. l : IdLnk
. . . . | . 15. lnk(kind(e)) = l
. . . . | . 16. t' : ℕ
```

```
. . . . | . 17. time(e) = t'
. . . . | . 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . | . 19. r<||snds(l;t')||
. . . . | . 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . | . 21. m : ℕt'
. . . . | . 22. ||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||≤r
. . . . | .     & r-||concat(firstn(m;map(λt1.
. . . . | .     m(l;t1);upto(t'))))||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . | .     & concat(map(λt1.m(l;t1);upto(t')))[r]
. . . . | .       =
. . . . | .       map(λt1.m(l;t1);upto(t'))[m][(r
. . . . | .       -||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||)]
. . . . | . ⊢  m<t'  by Auto
. . . . |
. . . . .\
. . . . . 1. the_w : World
. . . . . 2. e : E
. . . . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . . 4. ∀i:Id, t:ℕ.
. . . . . |  isnull(a(i;t))
. . . . . |  ⇒
. . . . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . |  isrcv(l;a(i;t))
. . . . . |  ⇒
. . . . . |  destination(l) = i
. . . . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . 6. ∀l:IdLnk, t:ℕ.
. . . . . |  ∃t':ℕ.
. . . . . |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . 7. isrcv(kind(e))
. . . . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . 9. destination(lnk(kind(e))) = loc(e)
. . . . . 10. r : ℕ
. . . . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . . 12. ms : Msg
. . . . . 13. msg(a(loc(e);time(e))) = ms
. . . . . 14. l : IdLnk
. . . . . 15. lnk(kind(e)) = l
. . . . . 16. t' : ℕ
. . . . . 17. time(e) = t'
. . . . . 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . . 19. r<||snds(l;t')||
. . . . . 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . . 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . 22. ||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||≤r
. . . . . |   & r-||concat(firstn(m;map(λt1.
. . . . . |   m(l;t1);upto(t'))))||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . . |   & concat(map(λt1.m(l;t1);upto(t')))[r]
. . . . . |     =
. . . . . |     map(λt1.m(l;t1);upto(t'))[m][(r
. . . . . |     -||concat(firstn(m;map(λt1.m(l;t1);upto(t'))))||)]
. . . . . 23. m<t'
. . . . . ⊢  ∃t:ℕt'.
```

```
. . . . . |  ||snds(l;t)||≤r
. . . . . |  & r<||snds(l;t)||+||onlnk(l;m(source(l);t))||
. . . . . | by RWO
. . . . . |    Thm* ∀f:(Top→Top), n:ℕ, l:Top List.
. . . . . |         firstn(n;map(f;l)) ~ map(f;firstn(n;l))
. . . . . |    {firstn_map}
. . . . . |    -2
. . . . . |    THEN
. . . . . |    RWO
. . . . . |    Thm* ∀n,m:ℕ.
. . . . . |         firstn(n;upto(m)) ~ if n≤_2m→ upto(n)
. . . . . |                             else upto(m) fi
. . . . . |    {firstn_upto}
. . . . . |    -2
. . . . . |    THEN
. . . . . |    Subst ((m≤_2t') ~ true_2) -2
. . . . . |    THEN
. . . . . |    Try (Complete (AutoBoolCase (m≤_2t')))
. . . . . |    THEN
. . . . . |    Reduce -2
. . . . . |    THEN
. . . . . |    Fold `w-snds` -2
. . . . . |    THEN
. . . . . |    ExRepD
. . . . . |
. . . . . 1. the_w : World
. . . . . 2. e : E
. . . . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . . 4. ∀i:Id, t:ℕ.
. . . . . |  isnull(a(i;t))
. . . . . |  ⇒
. . . . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . |  isrcv(l;a(i;t))
. . . . . |  ⇒
. . . . . |  destination(l) = i
. . . . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . 6. ∀l:IdLnk, t:ℕ.
. . . . . |  ∃t':ℕ.
. . . . . |  t≤t' & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . 7. isrcv(kind(e))
. . . . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . 9. destination(lnk(kind(e))) = loc(e)
. . . . . 10. r : ℕ
. . . . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . . 12. ms : Msg
. . . . . 13. msg(a(loc(e);time(e))) = ms
. . . . . 14. l : IdLnk
. . . . . 15. lnk(kind(e)) = l
. . . . . 16. t' : ℕ
. . . . . 17. time(e) = t'
. . . . . 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . . 19. r<||snds(l;t')||
. . . . . 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
```

```
. . . . . 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . 22. ||snds(l;m)||≤r
. . . . . 23. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . . 24. snds(l;t')[r]
. . . . . |    =
. . . . . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . . 25. m<t'
. . . . . ⊢  ∃t:ℕt'.
. . . . . |  ||snds(l;t)||≤r
. . . . . |  & r<||snds(l;t)||+||onlnk(l;m(source(l);t))||
. . . . . | by InstConcl [m]
. . . . . |
. . . . . |\
. . . . . | 1. the_w : World
. . . . . | 2. e : E
. . . . . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . . | 4. ∀i:Id, t:ℕ.
. . . . . | |  isnull(a(i;t))
. . . . . | |  ⇒
. . . . . | |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | |  isrcv(l;a(i;t))
. . . . . | |  ⇒
. . . . . | |  destination(l) = i
. . . . . | |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . | 6. ∀l:IdLnk, t:ℕ.
. . . . . | |  ∃t':ℕ.
. . . . . | |  t≤t'
. . . . . | |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . | 7. isrcv(kind(e))
. . . . . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . | 9. destination(lnk(kind(e))) = loc(e)
. . . . . | 10. r : ℕ
. . . . . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . . | 12. ms : Msg
. . . . . | 13. msg(a(loc(e);time(e))) = ms
. . . . . | 14. l : IdLnk
. . . . . | 15. lnk(kind(e)) = l
. . . . . | 16. t' : ℕ
. . . . . | 17. time(e) = t'
. . . . . | 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . . | 19. r<||snds(l;t')||
. . . . . | 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . . | 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . | 22. ||snds(l;m)||≤r
. . . . . | 23. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . . | 24. snds(l;t')[r]
. . . . . | |    =
. . . . . | |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . . | 25. m<t'
. . . . . | ⊢  r<||snds(l;m)||+||onlnk(l;m(source(l);m))||
. . . . . | | by RWO
. . . . . | |    Thm* ∀f:(A→B), as:A List, n:ℕ||as||.
. . . . . | |         map(f;as)[n] = f(as[n])
```

```
. . . . . | |    {map_select}
. . . . . | |    -3
. . . . . | |
. . . . . | |\
. . . . . | | 1. the_w : World
. . . . . | | 2. e : E
. . . . . | | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . . | | 4. ∀i:Id, t:ℕ.
. . . . . | |    isnull(a(i;t))
. . . . . | |    ⇒
. . . . . | |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . | | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | |    isrcv(l;a(i;t))
. . . . . | |    ⇒
. . . . . | |    destination(l) = i
. . . . . | |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . | | 6. ∀l:IdLnk, t:ℕ.
. . . . . | |    ∃t':ℕ.
. . . . . | |    t≤t'
. . . . . | |    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . | | 7. isrcv(kind(e))
. . . . . | | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . | | 9. destination(lnk(kind(e))) = loc(e)
. . . . . | | 10. r : ℕ
. . . . . | | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . . | | 12. ms : Msg
. . . . . | | 13. msg(a(loc(e);time(e))) = ms
. . . . . | | 14. l : IdLnk
. . . . . | | 15. lnk(kind(e)) = l
. . . . . | | 16. t' : ℕ
. . . . . | | 17. time(e) = t'
. . . . . | | 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . . | | 19. r<||snds(l;t')||
. . . . . | | 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . . | | 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . | | 22. ||snds(l;m)||≤r
. . . . . | | 23. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . . | | 24. snds(l;t')[r]
. . . . . | |     =
. . . . . | |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . . | | 25. m<t'
. . . . . | | ⊢  m<||upto(t')||
. . . . . | |   by RWO Thm* ∀n:ℕ. ||upto(n)|| ~ n    {length_upto} 0
. . . . . | |
. . . . . | .\
. . . . . | . 1. the_w : World
. . . . . | . 2. e : E
. . . . . | . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . . | . 4. ∀i:Id, t:ℕ.
. . . . . | . |  isnull(a(i;t))
. . . . . | . |  ⇒
. . . . . | . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . | . 5. ∀i:Id, t:ℕ, l:IdLnk.
```

```
. . . . . | . |  isrcv(l;a(i;t))
. . . . . | . |  ⇒
. . . . . | . |  destination(l) = i
. . . . . | . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . | . 6. ∀l:IdLnk, t:ℕ.
. . . . . | . |  ∃t':ℕ.
. . . . . | . |  t≤t'
. . . . . | . |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . | . 7. isrcv(kind(e))
. . . . . | . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . | . 9. destination(lnk(kind(e))) = loc(e)
. . . . . | . 10. r : ℕ
. . . . . | . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . . | . 12. ms : Msg
. . . . . | . 13. msg(a(loc(e);time(e))) = ms
. . . . . | . 14. l : IdLnk
. . . . . | . 15. lnk(kind(e)) = l
. . . . . | . 16. t' : ℕ
. . . . . | . 17. time(e) = t'
. . . . . | . 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . . | . 19. r<||snds(l;t')||
. . . . . | . 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . . | . 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . | . 22. ||snds(l;m)||≤r
. . . . . | . 23. r-||snds(l;m)||<||(λt1.m(l;t1))(upto(t')[m])||
. . . . . | . 24. snds(l;t')[r]
. . . . . | . |   =
. . . . . | . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . . | . 25. m<t'
. . . . . | . ⊢  r<||snds(l;m)||+||onlnk(l;m(source(l);m))||
. . . . . | . | by Reduce -3 THEN Subst' (upto(t')[m] = m) -3
. . . . . | . |

. . . . . | . |\
. . . . . | . | 1. the_w : World
. . . . . | . | 2. e : E
. . . . . | . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | . |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . . | . | 4. ∀i:Id, t:ℕ.
. . . . . | . |    isnull(a(i;t))
. . . . . | . |    ⇒
. . . . . | . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . | . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | . |    isrcv(l;a(i;t))
. . . . . | . |    ⇒
. . . . . | . |    destination(l) = i
. . . . . | . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . | . | 6. ∀l:IdLnk, t:ℕ.
. . . . . | . |    ∃t':ℕ.
. . . . . | . |    t≤t'
. . . . . | . |    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . | . | 7. isrcv(kind(e))
. . . . . | . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . | . | 9. destination(lnk(kind(e))) = loc(e)
. . . . . | . | 10. r : ℕ
. . . . . | . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
```

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. . . . . | . | 12. ms : Msg
. . . . . | . | 13. msg(a(loc(e);time(e))) = ms
. . . . . | . | 14. l : IdLnk
. . . . . | . | 15. lnk(kind(e)) = l
. . . . . | . | 16. t' : ℕ
. . . . . | . | 17. time(e) = t'
. . . . . | . | 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . . | . | 19. r<||snds(l;t')||
. . . . . | . | 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . . | . | 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . | . | 22. ||snds(l;m)||≤r
. . . . . | . | 23. r-||snds(l;m)||<||m(l;upto(t')[m])||
. . . . . | . | 24. snds(l;t')[r]
. . . . . | . |     =
. . . . . | . |     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . . | . | 25. m<t'
. . . . . | . | ⊢  m<||upto(t')||
. . . . . | . |   by RWO Thm* ∀n:ℕ. ||upto(n)|| ~ n    {length_upto}
. . . . . | . |     0
. . . . . | . |
. . . . . | . |\
. . . . . | . | 1. the_w : World
. . . . . | . | 2. e : E
. . . . . | . | 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | . |    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . . | . | 4. ∀i:Id, t:ℕ.
. . . . . | . |    isnull(a(i;t))
. . . . . | . |    ⇒
. . . . . | . |    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . | . | 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | . |    isrcv(l;a(i;t))
. . . . . | . |    ⇒
. . . . . | . |    destination(l) = i
. . . . . | . |    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . | . | 6. ∀l:IdLnk, t:ℕ.
. . . . . | . |    ∃t':ℕ.
. . . . . | . |    t≤t'
. . . . . | . |    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . | . | 7. isrcv(kind(e))
. . . . . | . | 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . | . | 9. destination(lnk(kind(e))) = loc(e)
. . . . . | . | 10. r : ℕ
. . . . . | . | 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . . | . | 12. ms : Msg
. . . . . | . | 13. msg(a(loc(e);time(e))) = ms
. . . . . | . | 14. l : IdLnk
. . . . . | . | 15. lnk(kind(e)) = l
. . . . . | . | 16. t' : ℕ
. . . . . | . | 17. time(e) = t'
. . . . . | . | 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . . | . | 19. r<||snds(l;t')||
. . . . . | . | 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . . | . | 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . | . | 22. ||snds(l;m)||≤r
. . . . . | . | 23. r-||snds(l;m)||<||m(l;upto(t')[m])||
```

```
. . . . . | . | 24. snds(l;t')[r]
. . . . . | . |      =
. . . . . | . |      map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . . | . | 25. m<t'
. . . . . | . | ⊢  upto(t')[m] = m
. . . . . | . |   by BackThru
. . . . . | . |      Thm* ∀m:ℕ, n:ℕm. upto(m)[n] = n    {select_upto}
. . . . . | . |
. . . . . | . .\
. . . . . | . . 1. the_w : World
. . . . . | . . 2. e : E
. . . . . | . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | . . |  ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . . | . . 4. ∀i:Id, t:ℕ.
. . . . . | . . |  isnull(a(i;t))
. . . . . | . . |  ⇒
. . . . . | . . |  (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . | . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | . . |  isrcv(l;a(i;t))
. . . . . | . . |  ⇒
. . . . . | . . |  destination(l) = i
. . . . . | . . |  & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . | . . 6. ∀l:IdLnk, t:ℕ.
. . . . . | . . |  ∃t':ℕ.
. . . . . | . . |  t≤t'
. . . . . | . . |  & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . | . . 7. isrcv(kind(e))
. . . . . | . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . | . . 9. destination(lnk(kind(e))) = loc(e)
. . . . . | . . 10. r : ℕ
. . . . . | . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . . | . . 12. ms : Msg
. . . . . | . . 13. msg(a(loc(e);time(e))) = ms
. . . . . | . . 14. l : IdLnk
. . . . . | . . 15. lnk(kind(e)) = l
. . . . . | . . 16. t' : ℕ
. . . . . | . . 17. time(e) = t'
. . . . . | . . 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . . | . . 19. r<||snds(l;t')||
. . . . . | . . 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . . | . . 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . | . . 22. ||snds(l;m)||≤r
. . . . . | . . 23. r-||snds(l;m)||<||m(l;m)||
. . . . . | . . 24. snds(l;t')[r]
. . . . . | . . |   =
. . . . . | . . |   map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . . | . . 25. m<t'
. . . . . | . . ⊢  r<||snds(l;m)||+||onlnk(l;m(source(l);m))||
. . . . . | . . | by Unfold `w-ml` -3
. . . . . | . . |
. . . . . | . . .....subterm .....
. . . . . | . . 1. the_w : World
. . . . . | . . 2. e : E
. . . . . | . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . | . .    ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
```

```
. . . . . . | . . 4. ∀i:Id, t:ℕ.
. . . . . . | . .    isnull(a(i;t))
. . . . . . | . .    ⇒
. . . . . . | . .    (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . . | . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . . | . .    isrcv(l;a(i;t))
. . . . . . | . .    ⇒
. . . . . . | . .    destination(l) = i
. . . . . . | . .    & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . . | . . 6. ∀l:IdLnk, t:ℕ.
. . . . . . | . .    ∃t':ℕ.
. . . . . . | . .    t≤t'
. . . . . . | . .    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . . | . . 7. isrcv(kind(e))
. . . . . . | . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . . | . . 9. destination(lnk(kind(e))) = loc(e)
. . . . . . | . . 10. r : ℕ
. . . . . . | . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . . . | . . 12. ms : Msg
. . . . . . | . . 13. msg(a(loc(e);time(e))) = ms
. . . . . . | . . 14. l : IdLnk
. . . . . . | . . 15. lnk(kind(e)) = l
. . . . . . | . . 16. t' : ℕ
. . . . . . | . . 17. time(e) = t'
. . . . . . | . . 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . . . | . . 19. r<||snds(l;t')||
. . . . . . | . . 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . . . | . . 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . . | . . 22. ||snds(l;m)||≤r
. . . . . . | . . 23. r-||snds(l;m)||<||onlnk(l;m(source(l);m))||
. . . . . . | . . 24. snds(l;t')[r]
. . . . . . | . .     =
. . . . . . | . .     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . . . | . . 25. m<t'
. . . . . . | . . ⊢  ||onlnk(l;m(source(l);m))|| ∈ ℤ
. . . . . . | . .   by BackThruSomeHyp
. . . . . . |
. . . . . . .\
. . . . . . . .....subterm .....
. . . . . . . 1. the_w : World
. . . . . . . 2. e : E
. . . . . . . 3. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . .      ¬source(l) = i ⇒ onlnk(l;m(i;t)) = nil
. . . . . . . 4. ∀i:Id, t:ℕ.
. . . . . .      isnull(a(i;t))
. . . . . .      ⇒
. . . . . .      (∀x:Id. s(i;t+1).x = s(i;t).x) & m(i;t) = nil
. . . . . . . 5. ∀i:Id, t:ℕ, l:IdLnk.
. . . . . .      isrcv(l;a(i;t))
. . . . . .      ⇒
. . . . . .      destination(l) = i
. . . . . .      & ||queue(l;t)||≥1 & hd(queue(l;t)) = msg(a(i;t))
. . . . . . . 6. ∀l:IdLnk, t:ℕ.
. . . . . .      ∃t':ℕ.
. . . . . .      t≤t'
```

```
. . . . . .    & isrcv(l;a(destination(l);t')) ∨ queue(l;t') = nil
. . . . . . 7. isrcv(kind(e))
. . . . . . 8. isrcv(lnk(kind(e));a(loc(e);time(e)))
. . . . . . 9. destination(lnk(kind(e))) = loc(e)
. . . . . . 10. r : ℕ
. . . . . . 11. ||rcvs(lnk(kind(e));time(e))|| = r
. . . . . . 12. ms : Msg
. . . . . . 13. msg(a(loc(e);time(e))) = ms
. . . . . . 14. l : IdLnk
. . . . . . 15. lnk(kind(e)) = l
. . . . . . 16. t' : ℕ
. . . . . . 17. time(e) = t'

. . . . . . 18. ∀t:ℕt'. ||onlnk(l;m(source(l);t))|| ∈ ℤ
. . . . . . 19. r<||snds(l;t')||
. . . . . . 20. concat(map(λt1.m(l;t1);upto(t')))[r] = ms
. . . . . . 21. m : ℕ||map(λt1.m(l;t1);upto(t'))||
. . . . . . 22. ||snds(l;m)||≤r
. . . . . . 23. r-||snds(l;m)||<||map(λt1.m(l;t1);upto(t'))[m]||
. . . . . . 24. snds(l;t')[r]
. . . . . .     =
. . . . . .     map(λt1.m(l;t1);upto(t'))[m][(r-||snds(l;m)||)]
. . . . . . 25. m<t'
. . . . . . 26. t : ℕt'
. . . . . . ⊢  ||onlnk(l;m(source(l);t))|| ∈ ℤ  by BackThruSomeHyp
END
```